

Calendar FAQ, v. 2.7 (modified 15 January 2005) Part 1/3

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FREQUENTLY ASKED QUESTIONS ABOUT CALENDARS Part 1 of 3

Version 2.7 – 15 January 2005

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Introduction

This is the calendar FAQ. Its purpose is to give an overview of the Christian, Hebrew, Persian, and Islamic calendars in common use. It will provide a historical background for the Christian calendar, plus an overview of the French Revolutionary calendar, the Maya calendar, and the Chinese calendar.

Comments are very welcome. My e-mail address is given above.

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Changes in version 2.7

Section 2.5 added and the following sections renumbered.
Section 2.13.8 added and the following sections renumbered.
The description of the Venerable Bede in section 2.14 has been
changed.
A clarification has been added to section 3.8.
The rules in section 4.5 have been brought up to date.
A few minor corrections have been made.

Writing dates and years

Dates will be written in the British format (1 January)
rather than the American format (January 1). Dates will
occasionally be abbreviated: "1 Jan" rather than "1 January".

Years before and after the "official" birth year of Christ
will be written "45 BC" or "AD 1997", respectively. I prefer
this notation over the secular "45 BCE" and "1997 CE"
(See also section 2.13.4.)

The 'mod' operator

Throughout this document the operator 'mod' will be used to
signify the modulo or remainder operator. For example,
 $17 \bmod 7 = 3$ because the result of the division $17/7$ is 2 with a
remainder of 3.

The text in square brackets

Square brackets [like this] identify information that I am unsure about and about which I would like more information. Please write me at claus@tondering.dk (and please include the word "calendar" in the subject line).

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1. What Astronomical Events Form the Basis of Calendars?

Calendars are normally based on astronomical events, and the two most important astronomical objects are the sun and the moon. Their cycles are very important in the construction and understanding of calendars.

Our concept of a year is based on the earth's motion around the sun. The time from one fixed point, such as a solstice or equinox, to the next is called a "tropical year". Its length is currently 365.242190 days, but it varies. Around 1900 its length was 365.242196 days, and around 2100 it will be 365.242184 days. (This definition of the tropical year is not quite accurate, see section 1.1 for more details.)

Our concept of a month is based on the moon's motion around the earth, although this connection has been broken in the calendar commonly used now. The time from one new moon to the next is called a "synodic month", and its length is currently 29.5305889 days, but it varies. Around 1900 its length was 29.5305886 days, and around 2100 it will be 29.5305891 days.

Note that these numbers are averages. The actual length of a particular year may vary by several minutes due to the influence of the gravitational force from other planets. Similarly, the time

between two new moons may vary by several hours due to a number of factors, including changes in the gravitational force from the sun, and the moon's orbital inclination.

It is unfortunate that the length of the tropical year is not a multiple of the length of the synodic month. This means that with 12 months per year, the relationship between our month and the moon cannot be maintained.

However, 19 tropical years is 234.997 synodic months, which is very close to an integer. So every 19 years the phases of the moon fall on the same dates (if it were not for the skewness introduced by leap years). 19 years is called a Metonic cycle (after Meton, an astronomer from Athens in the 5th century BC).

So, to summarise: There are three important numbers to note:
A tropical year is 365.24219 days.
A synodic month is 29.53059 days.
19 tropical years is close to an integral number of synodic months.

The Christian calendar is based on the motion of the earth around the sun, while the months retain no connection with the motion of the moon.

On the other hand, the Islamic calendar is based on the motion of the moon, while the year has no connection with the motion of the earth around the sun.

Finally, the Hebrew calendar combines both, in that its years are linked to the motion of the earth around the sun, and its months are linked to the motion of the moon.

1.1. What are equinoxes and solstices?

Equinoxes and solstices are frequently used as anchor points for calendars. For people in the northern hemisphere:

- Winter solstice is the time in December when the sun reaches its southernmost latitude. At this time we have the shortest day. The date is near 21 December.
- Summer solstice is the time in June when the sun reaches its northernmost latitude. At this time we have the longest day. The date is near 21 June.
- Vernal equinox is the time in March when the sun passes the equator moving from the southern to the northern hemisphere. Day and night have approximately the same length. The date is near 20 March.
- Autumnal equinox is the time in September when the sun passes the equator moving from the northern to the southern hemisphere. Day and

night have approximately the same length. The date is near 22 September.

For people in the southern hemisphere these events are shifted half a year.

The astronomical "tropical year" is frequently defined as the time between, say, two vernal equinoxes, but this is not actually true. Currently the time between two vernal equinoxes is slightly greater than the tropical year. The reason is that the earth's position in its orbit at the time of solstices and equinoxes shifts slightly each year (taking approximately 21,000 years to move all the way around the orbit). This, combined with the fact that the earth's orbit is not completely circular, causes the equinoxes and solstices to shift with respect to each other.

The astronomer's mean tropical year is really a somewhat artificial average of the period between the time when the sun is in any given position in the sky with respect to the equinoxes and the next time the sun is in the same position.

2. The Christian Calendar

The "Christian calendar" is the term traditionally used to designate the calendar commonly in use, although its connection with Christianity is highly debatable.

The Christian calendar has years of 365 or 366 days. It is divided into 12 months that have no relationship to the motion of the moon. In parallel with this system, the concept of "weeks" groups the days in sets of 7.

Two main versions of the Christian calendar have existed in recent times: The Julian calendar and the Gregorian calendar. The difference between them lies in the way they approximate the length of the tropical year and their rules for calculating Easter.

2.1. What is the Julian calendar?

The Julian calendar was introduced by Julius Caesar in 45 BC. It was in common use until the late 1500s, when countries started changing to the Gregorian calendar (section 2.2). However, some countries (for example, Greece and Russia) used it into the early 1900s, and the Orthodox church in Russia still uses it, as do some other Orthodox churches.

In the Julian calendar, the tropical year is approximated as $365 \frac{1}{4}$ days = 365.25 days. This gives an error of 1 day in approximately 128 years.

The approximation $365 \frac{1}{4}$ is achieved by having 1 leap year every 4 years.

2.1.1. What years are leap years?

The Julian calendar has 1 leap year every 4 years:

Every year divisible by 4 is a leap year.

However, the 4-year rule was not followed in the first years after the introduction of the Julian calendar in 45 BC. Due to a counting error, every 3rd year was a leap year in the first years of this calendar's existence. The leap years were:

45 BC(?), 42 BC, 39 BC, 36 BC, 33 BC, 30 BC,
27 BC, 24 BC, 21 BC, 18 BC, 15 BC, 12 BC, 9 BC,
AD 8, AD 12, and every 4th year from then on.

Authorities disagree about whether 45 BC was a leap year or not.

There were no leap years between 9 BC and AD 8 (or, according to some authorities, between 12 BC and AD 4). This period without leap years was decreed by emperor Augustus in order to make up for the surplus of leap years introduced previously, and it earned him a place in the calendar as the 8th month was named after him.

It is a curious fact that although the method of reckoning years after the (official) birthyear of Christ was not introduced until the 6th century, by some stroke of luck the Julian leap years coincide with years of our Lord that are divisible by 4.

2.1.2. What consequences did the use of the Julian calendar have?

The Julian calendar introduces an error of 1 day every 128 years. So every 128 years the tropical year shifts one day backwards with respect to the calendar. Furthermore, the method for calculating the dates for Easter was inaccurate and needed to be refined.

In order to remedy this, two steps were necessary: 1) The Julian calendar had to be replaced by something more adequate. 2) The extra days that the Julian calendar had inserted had to be dropped.

The solution to problem 1) was the Gregorian calendar described in section 2.2.

The solution to problem 2) depended on the fact that it was felt that 21 March was the proper day for vernal equinox (because 21 March was the date for vernal equinox during the Council of Nicaea in AD 325). The Gregorian calendar was therefore calibrated to make that day

vernal equinox.

By 1582 vernal equinox had moved $(1582-325)/128$ days = approximately 10 days backwards. So 10 days had to be dropped.

2.2. What is the Gregorian calendar?

The Gregorian calendar is the one commonly used today. It was proposed by Aloysius Lilius, a physician from Naples, and adopted by Pope Gregory XIII in accordance with instructions from the Council of Trent (1545–1563) to correct for errors in the older Julian Calendar. It was decreed by Pope Gregory XIII in a papal bull on 24 February 1582. This bull is named "Inter Gravissimas" after its first two words.

In the Gregorian calendar, the tropical year is approximated as $365 \frac{97}{400}$ days = 365.2425 days. Thus it takes approximately 3300 years for the tropical year to shift one day with respect to the Gregorian calendar.

The approximation $365 \frac{97}{400}$ is achieved by having 97 leap years every 400 years.

2.2.1. What years are leap years?

The Gregorian calendar has 97 leap years every 400 years:

Every year divisible by 4 is a leap year.

However, every year divisible by 100 is not a leap year.

However, every year divisible by 400 is a leap year after all.

So, 1700, 1800, 1900, 2100, and 2200 are not leap years. But 1600, 2000, and 2400 are leap years.

(Destruction of a myth: There are no double leap years, i.e. no years with 367 days. See, however, the note on Sweden in section 2.2.4.)

2.2.2. Isn't there a 4000-year rule?

It has been suggested (by the astronomer John Herschel (1792–1871) among others) that a better approximation to the length of the tropical year would be $365 \frac{969}{4000}$ days = 365.24225 days. This would dictate 969 leap years every 4000 years, rather than the 970 leap years mandated by the Gregorian calendar. This could be achieved by dropping one leap year from the Gregorian calendar every 4000 years, which would make years divisible by 4000 non-leap years.

This rule has, however, not been officially adopted.

2.2.3. Don't the Greek do it differently?

When the Orthodox church in Greece finally decided to switch to the Gregorian calendar in the 1920s, they tried to improve on the Gregorian leap year rules, replacing the "divisible by 400" rule by the following:

Every year which when divided by 900 leaves a remainder of 200 or 600 is a leap year.

This makes 1900, 2100, 2200, 2300, 2500, 2600, 2700, 2800 non-leap years, whereas 2000, 2400, and 2900 are leap years. This will not create a conflict with the rest of the world until the year 2800.

This rule gives 218 leap years every 900 years, which gives us an average year of $365 \frac{218}{900}$ days = 365.24222 days, which is certainly more accurate than the official Gregorian number of 365.2425 days.

However, this rule is *not* official in Greece.

2.2.4. When did country X change from the Julian to the Gregorian calendar?

The papal bull of February 1582 decreed that 10 days should be dropped from October 1582 so that 15 October should follow immediately after 4 October, and from then on the reformed calendar should be used.

This was observed in Italy, Poland, Portugal, and Spain. Other Catholic countries followed shortly after, but Protestant countries were reluctant to change, and the Greek orthodox countries didn't change until the start of the 1900s.

Changes in the 1500s required 10 days to be dropped.
Changes in the 1600s required 10 days to be dropped.
Changes in the 1700s required 11 days to be dropped.
Changes in the 1800s required 12 days to be dropped.
Changes in the 1900s required 13 days to be dropped.

(Exercise for the reader: Why is the error in the 1600s the same as in the 1500s.)

The following list contains the dates for changes in a number of countries. It is very strange that in many cases there seems to be some doubt among authorities about what the correct days are. Different sources give very different dates in some cases. The list below does not include all the different opinions about when the change took place.

Albania: December 1912

Austria: Different regions on different dates

Brixen, Salzburg and Tyrol:

5 Oct 1583 was followed by 16 Oct 1583

Carinthia and Styria:

14 Dec 1583 was followed by 25 Dec 1583

See also Czechoslovakia and Hungary

Belgium: See the Netherlands

Bulgaria: 31 Mar 1916 was followed by 14 Apr 1916

Canada: Different areas changed at different times.

Newfoundland and Hudson Bay coast:

2 Sep 1752 was followed by 14 Sep 1752

Mainland Nova Scotia:

Gregorian 1605 – 13 Oct 1710

Julian 2 Oct 1710 – 2 Sep 1752

Gregorian since 14 Sep 1752

Rest of Canada:

Gregorian from first European settlement

China: The Gregorian calendar replaced the Chinese calendar in 1912, but the Gregorian calendar was not used throughout the country until the communist revolution of 1949.

Czechoslovakia (i.e. Bohemia and Moravia):

6 Jan 1584 was followed by 17 Jan 1584

Denmark (including Norway):

18 Feb 1700 was followed by 1 Mar 1700

Egypt: 1875

Estonia: 31 Jan 1918 was followed by 14 Feb 1918

Finland: Then part of Sweden. (Note, however, that Finland later became part of Russia, which then still used the Julian calendar. The Gregorian calendar remained official in Finland, but some use of the Julian calendar was made.)

France: 9 Dec 1582 was followed by 20 Dec 1582

Alsace: 5 Feb 1682 was followed by 16 Feb 1682

Lorraine: 16 Feb 1760 was followed by 28 Feb 1760

Strasbourg: February 1682

Germany: Different states on different dates:

Catholic states on various dates in 1583–1585

Prussia: 22 Aug 1610 was followed by 2 Sep 1610

Protestant states: 18 Feb 1700 was followed by 1 Mar 1700

(Many local variations)

Great Britain and Dominions:

2 Sep 1752 was followed by 14 Sep 1752

Greece: [9 Mar 1924 was followed by 23 Mar 1924

(Some sources say 1916 and 1920)]

Hungary: 21 Oct 1587 was followed by 1 Nov 1587

Ireland: See Great Britain

Italy: 4 Oct 1582 was followed by 15 Oct 1582

Japan: The Gregorian calendar was introduced to supplement the traditional Japanese calendar on 1 Jan 1873.

Latvia: During German occupation 1915 to 1918

Lithuania: 1915

Luxemburg: 14 Dec 1582 was followed by 25 Dec 1582

Netherlands (including Belgium):

Zeeland, Brabant, and the "Staten Generaal":

14 Dec 1582 was followed by 25 Dec 1582

Holland:

1 Jan 1583 was followed by 12 Jan 1583

Limburg and the southern provinces (currently Belgium):

20 Dec 1582 was followed by 31 Dec 1582

or

21 Dec 1582 was followed by 1 Jan 1583

Groningen:

10 Feb 1583 was followed by 21 Feb 1583

Went back to Julian in the summer of 1594

31 Dec 1700 was followed by 12 Jan 1701

Gelderland:

30 Jun 1700 was followed by 12 Jul 1700

Utrecht and Overijssel:

30 Nov 1700 was followed by 12 Dec 1700

Friesland:

31 Dec 1700 was followed by 12 Jan 1701

Drenthe:

30 Apr 1701 was followed by 12 May 1701

Norway: Then part of Denmark.

Poland: 4 Oct 1582 was followed by 15 Oct 1582

Portugal: 4 Oct 1582 was followed by 15 Oct 1582

Romania: 31 Mar 1919 was followed by 14 Apr 1919
[The Greek Orthodox parts of the country may have
changed later.]

Russia: 31 Jan 1918 was followed by 14 Feb 1918
[In the eastern parts of the country the change may
not have occurred until 1920.]

Scotland: Much confusion exists regarding Scotland's change. Different
authorities disagree about whether Scotland changed together
with the rest of Great Britain, or if they had changed
earlier.

Spain: 4 Oct 1582 was followed by 15 Oct 1582

Sweden (including Finland):
17 Feb 1753 was followed by 1 Mar 1753 (see note below)

Switzerland:
Catholic cantons: 1583, 1584 or 1597
Protestant cantons:
31 Dec 1700 was followed by 12 Jan 1701
(Many local variations)

Turkey: Gregorian calendar introduced 1 Jan 1927

United States: Different areas changed at different times.
Along the Eastern seaboard: With Great Britain in 1752.
Mississippi valley: With France in 1582.
Texas, Florida, California, Nevada, Arizona, New Mexico:
With Spain in 1582
Washington, Oregon: With Britain in 1752.
Alaska: October 1867 when Alaska became part of the USA.

Wales: See Great Britain

Yugoslavia: 1919

Sweden has a curious history. Sweden decided to make a gradual change
from the Julian to the Gregorian calendar. By dropping every leap year
from 1700 through 1740 the eleven superfluous days would be omitted
and from 1 Mar 1740 they would be in sync with the Gregorian
calendar. (But in the meantime they would be in sync with nobody!)

So 1700 (which should have been a leap year in the Julian calendar)
was not a leap year in Sweden. However, by mistake 1704 and 1708
became leap years. This left Sweden out of synchronisation with both
the Julian and the Gregorian world, so they decided to go *back* to
the Julian calendar. In order to do this, they inserted an extra day
in 1712, making that year a double leap year! So in 1712, February had
30 days in Sweden.

Later, in 1753, Sweden changed to the Gregorian calendar by dropping 11 days like everyone else.

2.3. What day is the leap day?

It is 24 February!

Weird? Yes! The explanation is related to the Roman calendar and is found in section 2.8.1.

>*From a numerical point of view, of course 29 February is the extra day. But from the point of view of celebration of feast days, the following correspondence between days in leap years and non-leap years has traditionally been used:*

Non-leap year	Leap year
22 February	22 February
23 February	23 February
	24 February (extra day)
24 February	25 February
25 February	26 February
26 February	27 February
27 February	28 February
28 February	29 February

For example, the feast of St. Leander has been celebrated on 27 February in non-leap years and on 28 February in leap years.

Many countries are gradually changing the leap day from the 24th to the 29th. This affects countries such as Sweden and Austria that celebrate "name days" (i.e. each day is associated with a name).

2.4. What is the Solar Cycle?

In the Julian calendar the relationship between the days of the week and the dates of the year is repeated in cycles of 28 years. In the Gregorian calendar this is still true for periods that do not cross years that are divisible by 100 but not by 400.

A period of 28 years is called a Solar Cycle. The "Solar Number" of a year is found as:

$$\text{Solar Number} = (\text{year} + 8) \bmod 28 + 1$$

In the Julian calendar there is a one-to-one relationship between the Solar Number and the day on which a particular date falls.

(The leap year cycle of the Gregorian calendar is 400 years, which is 146,097 days, which curiously enough is a multiple of 7. So in the Gregorian calendar the equivalent of the "Solar Cycle" would be 400 years, not $7*400=2800$ years as one might be tempted to believe.)

2.5. What is the Dominical Letter?

Each ordinary (non-leap) year is assigned a letter in the range A to G which describes what days of the year are Sundays. This letter is called the "Dominical Letter" ("Sunday Letter") of the year.

It works in this manner: Assign the letter A to 1 January, B to 2 Jan, C to 3 Jan, ... G to 7 Jan, A to 8 Jan, B to 9 Jan, and so on, using the letters A to G and omitting the leap day.

In a year with Dominical Letter A, all days marked A are Sundays. In a year with Dominical Letter B, all days marked B are Sundays. And so on.

Leap years have two Dominical Letters, one which is used from the start of January until the leap day, and another one which is used for the rest of the year.

The Dominical Letter of 2005 is B. The Dominical Letters of 2004 were D and C.

2.6. What day of the week was 2 August 1953?

To calculate the day on which a particular date falls, the following algorithm may be used (the divisions are integer divisions, in which remainders are discarded):

$$a = (14 - \text{month}) / 12$$

$$y = \text{year} - a$$

$$m = \text{month} + 12*a - 2$$

$$\text{For Julian calendar: } d = (5 + \text{day} + y + y/4 + (31*m)/12) \bmod 7$$

$$\text{For Gregorian calendar: } d = (\text{day} + y + y/4 - y/100 + y/400 + (31*m)/12) \bmod 7$$

The value of d is 0 for a Sunday, 1 for a Monday, 2 for a Tuesday, etc.

Example: On what day of the week was the author born?

My birthday is 2 August 1953 (Gregorian, of course).

$$a = (14 - 8) / 12 = 0$$

$$y = 1953 - 0 = 1953$$

$$m = 8 + 12*0 - 2 = 6$$

$$d = (2 + 1953 + 1953/4 - 1953/100 + 1953/400 + (31*6)/12) \bmod 7 \\ = (2 + 1953 + 488 - 19 + 4 + 15) \bmod 7$$

$$\begin{aligned} &= 2443 \bmod 7 \\ &= 0 \end{aligned}$$

I was born on a Sunday.

2.7. When can I reuse my 1992 calendar?

Let us first assume that you are only interested in which dates fall on which days of the week; you are not interested in the dates for Easter and other irregular holidays.

Let us further confine ourselves to the years 1901–2099.

With these restrictions, the answer is as follows:

- If year X is a leap year, you can reuse its calendar in year $X+28$.
- If year X is the first year after a leap year, you can reuse its calendar in years $X+6$, $X+17$, and $X+28$.
- If year X is the second year after a leap year, you can reuse its calendar in years $X+11$, $X+17$, and $X+28$.
- If year X is the third year after a leap year, you can reuse its calendar in years $X+11$, $X+22$, and $X+28$.

Note that the expression $X+28$ occurs in all four items above. So you can always reuse your calendar every 28 years.

But if you also want your calendar's indication of Easter and other Christian holidays to be correct, the rules are far too complex to be put to a simple formula. Sometimes calendars can be reused after just six years. For example, the calendars for the years 1981 and 1987 are identical, even when it comes to the date for Easter. But sometimes a very long time can pass before a calendar can be reused; if you happen to have a calendar from 1940, you won't be able to reuse it until the year 5280!

2.8. What is the Roman calendar?

Before Julius Caesar introduced the Julian calendar in 45 BC, the Roman calendar was a mess, and much of our so-called "knowledge" about it seems to be little more than guesswork.

Originally, the year started on 1 March and consisted of only 304 days or 10 months (Martius, Aprilis, Maius, Junius, Quintilis, Sextilis, September, October, November, and December). These 304 days were followed by an unnamed and unnumbered winter period. The Roman king Numa Pompilius (c. 715–673 BC, although his historicity is disputed)

allegedly introduced February and January (in that order) between December and March, increasing the length of the year to 354 or 355 days. In 450 BC, February was moved to its current position between January and March.

In order to make up for the lack of days in a year, an extra month, Intercalaris or Mercedonius, (allegedly with 22 or 23 days though some authorities dispute this) was introduced in some years. In an 8 year period the length of the years were:

- 1: 12 months or 355 days
- 2: 13 months or 377 days
- 3: 12 months or 355 days
- 4: 13 months or 378 days
- 5: 12 months or 355 days
- 6: 13 months or 377 days
- 7: 12 months or 355 days
- 8: 13 months or 378 days

A total of 2930 days corresponding to a year of 366 1/4 days. This year was discovered to be too long, and therefore 7 days were later dropped from the 8th year, yielding 365.375 days per year.

This is all theory. In practice it was the duty of the priesthood to keep track of the calendars, but they failed miserably, partly due to ignorance, partly because they were bribed to make certain years long and other years short. Furthermore, leap years were considered unlucky and were therefore avoided in time of crisis, such as the Second Punic War.

In order to clean up this mess, Julius Caesar made his famous calendar reform in 45 BC. We can make an educated guess about the length of the months in the years 47 and 46 BC:

	47 BC	46 BC
January	29	29
February	28	24
Intercalaris	27	
March	31	31
April	29	29
May	31	31
June	29	29
Quintilis	31	31
Sextilis	29	29
September	29	29
October	31	31
November	29	29
Undecember	33	
Duodecember	34	
December	29	29
	---	---
Total	355	445

The length of the months from 45 BC onward were the same as the ones we know today.

Occasionally one reads the following story:

"Julius Caesar made all odd numbered months 31 days long, and all even numbered months 30 days long (with February having 29 days in non-leap years). In 44 BC Quintilis was renamed 'Julius' (July) in honour of Julius Caesar, and in 8 BC Sextilis became 'Augustus' in honour of emperor Augustus. When Augustus had a month named after him, he wanted his month to be a full 31 days long, so he removed a day from February and shifted the length of the other months so that August would have 31 days."

This story, however, has no basis in actual fact. It is a fabrication possibly dating back to the 14th century.

2.8.1. How did the Romans number days?

The Romans didn't number the days sequentially from 1. Instead they had three fixed points in each month:

"Kalendae" (or "Calendae"), which was the first day of the month.

"Idus", which was the 13th day of January, February, April, June, August, September, November, and December, or the 15th day of March, May, July, or October.

"Nonae", which was the 9th day before Idus (counting Idus itself as the 1st day).

The days between Kalendae and Nonae were called "the 5th day before Nonae", "the 4th day before Nonae", "the 3rd day before Nonae", and "the day before Nonae". (There was no "2nd day before Nonae". This was because of the inclusive way of counting used by the Romans: To them, Nonae itself was the first day, and thus "the 2nd day before" and "the day before" would mean the same thing.)

Similarly, the days between Nonae and Idus were called "the Xth day before Idus", and the days after Idus were called "the Xth day before Kalendae (of the next month)".

Julius Caesar decreed that in leap years the "6th day before Kalendae of March" should be doubled. So in contrast to our present system, in which we introduce an extra date (29 February), the Romans had the same date twice in leap years. The doubling of the 6th day before Kalendae of March is the origin of the word "bissextile". If we create a list of equivalences between the Roman days and our current days of February in a leap year, we get the following:

7th day before Kalendae of March 23 February
6th day before Kalendae of March 24 February
6th day before Kalendae of March 25 February
5th day before Kalendae of March 26 February

4th day before Kalendae of March 27 February
3rd day before Kalendae of March 28 February
the day before Kalendae of March 29 February
Kalendae of March 1 March

You can see that the extra 6th day (going backwards) falls on what is today 24 February. For this reason 24 February is still today considered the "extra day" in leap years (see section 2.3). However, at certain times in history the second 6th day (25 Feb) has been considered the leap day.

Why did Caesar choose to double the 6th day before Kalendae of March? It appears that the leap month Intercalaris/Mercedonius of the pre-reform calendar was not placed after February, but inside it, namely between the 7th and 6th day before Kalendae of March. It was therefore natural to have the leap day in the same position.

2.9. What is the proleptic calendar?

The Julian calendar was introduced in 45 BC, but when historians date events prior to that year, they normally extend the Julian calendar backward in time. This extended calendar is known as the "Julian Proleptic Calendar".

Similarly, it is possible to extend the Gregorian calendar backward in time before 1582. However, this "Gregorian Proleptic Calendar" is rarely used.

If someone refers to, for example, 15 March 429 BC, they are probably using the Julian proleptic calendar.

In the Julian proleptic calendar, year X BC is a leap year, if X-1 is divisible by 4. This is the natural extension of the Julian leap year rules.

2.10. Has the year always started on 1 January?

For the man in the street, yes. When Julius Caesar introduced his calendar in 45 BC, he made 1 January the start of the year, and it was always the date on which the Solar Number and the Golden Number (see section 2.13.3) were incremented.

However, the church didn't like the wild parties that took place at the start of the new year, and in AD 567 the council of Tours declared that having the year start on 1 January was an ancient mistake that should be abolished.

Through the middle ages various New Year dates were used. If an ancient document refers to year X, it may mean any of 7 different

periods in our present system:

- 1 Mar X to 28/29 Feb X+1
- 1 Jan X to 31 Dec X
- 1 Jan X-1 to 31 Dec X-1
- 25 Mar X-1 to 24 Mar X
- 25 Mar X to 24 Mar X+1
- Saturday before Easter X to Friday before Easter X+1
- 25 Dec X-1 to 24 Dec X

Choosing the right interpretation of a year number is difficult, so much more as one country might use different systems for religious and civil needs.

The Byzantine Empire used a year starting on 1 Sep, but they didn't count years since the birth of Christ, instead they counted years since the creation of the world which they dated to 1 September 5509 BC.

Since about 1600 most countries have used 1 January as the first day of the year. Italy and England, however, did not make 1 January official until around 1750.

In England (but not Scotland) three different years were used:

- The historical year, which started on 1 January.
- The liturgical year, which started on the first Sunday in advent.
- The civil year, which
from the 7th to the