Re: Stable Orbit Formulae?

Source: http://sci.tech−archive.net/Archive/sci.astro/2008−01/msg00057.html

- *From*: Timothy Partee <tpartee@xxxxxxxxxxxxx>
- *Date*: Wed, 09 Jan 2008 11:35:42 −0800

Martin Brown wrote:

> On Jan 4, 1:46 am, Timothy Partee <tpar...@xxxxxxxxxxxxx> wrote:
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>> Greg Neill wrote:
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>>> "Timothy Partee" <tpar...@xxxxxxxxxxxxx> wrote in message
>>> news:c7WdnY4sp7RM8uDanZ2dnUVZ_ramnZ2d@xxxxxxxxxxxxxxx
>>>
>>>> Given two (planetesimal) orbital bodies
>>>> whose Mass << Stellar Body,
>>>> what is the minimum permissible distance
>>>> between the two bodies such
>>>> that they will not disrupt one another's
>>>> orbital parameters?
>>>
>>> There will always be perturbations due to mutual interactions causing changes to the orbital parameters. Whether or not these perturbations can lead to disruptive changes (collisions or ejections) can be difficult to determine because it can depend upon resonances in the periods of the bodies which can amplify, damp, or induce cyclical changes.
>>> You might want to try a search on orbital resonances and peruse the topic.
>>
>> Hmmm, interesting. However, it would seem that our own solar system's planets have a negligible effect upon each other in their (nearly) concentric orbits around Sol. There must be a threshold of
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> We still can't prove long term stability though many have tried.
>
>> distance/mass (Gravitational Force) at which effects of one orbital body become negligible to the other. In stellar formation modeling this equation, threshold, formula or what−have−you must exist and be defined

There is nothing so simple that will work. The closest formula to meeting your requirements in general is Ovendens principle (which is anyway more of a conjecture) and states that given enough time planets will evolve into orbits where they avoid mutual interaction (or get flung out of the solar system). His "principle of least interaction" so you have to minimise the time average of his mutual interaction expression:

<R> = sum i,j (m[i]m[j]/(r[i]−r[j])) (i != j)

Whether or not you beieve this formula to be correct (AFAIK noone has proved it) it seems to roughly predict the right sort of behaviour in numerical simulations. A brief discussion of it exists in A E Roys book orbital motion.

There are basically no easy answers. Some systems which intuitively you might expect to be stable are unstable and vice versa. A discussion of one part of the restricted 3 body problem with some exploration of the regions of stability is online at:

http://www.iop.org/EJ/article/1538−3881/124/4/2332/202196.text.html

Most of the others require subscription access. This has some charts for the regions of stability obtained by numerical simulation for one restricted version of the 1:1 resonance problem.

> I'm just looking for a general and believable "magic number" that says "orbital bodies A and B's semi−major axii should be at least X km distant from one another given A, B and stellar masses and assuming a perfectly circular, flat, non−eccentric orbits".

Nothing quite like that exists either, although again you could conjecture that a variant of Bodes law might give you a sporting chance of arranging a toy solar system that will not fall apart too easily under perturbations. Several orbital simulators exist on the web and you could try out a few ideas in one of those.

Even resonances can be good or bad. Jupiter and Saturn appear to be roughly phase locked 5:2 on orbital period. Pluto and Neptune by 2:3 (and appear stable despite the fact that Pluto is in an eccentric orbit that goes inside Neptunes more circular path). And 3 of the Trojan moons of Jupiter are locked together in a 3:2:1 stable resonance.

> hugely ignorant on my part of some peculiarity of orbital physics... Which is the whole reason I'm asking the question here. =)

> You have asked one of those simple sounding questions for which no simple answer exists.
>
> There are research papers around on orbital stability proofs from an analytical perspective but they are incomprehensible to all but specialists in the field. Numerical simulations are more accessible if you can find them. If you are serious about this I'd recommend A E Roys Orbital Motion for an introduction (warning not an easy read)
>
> Regards,
> Martin Brown

Thank you for the references and additional background every little bit helps. =) To be honest this is all for an ultra−realistic online game that I'm working on with the help of some friends that is themed around "galaxy exploration and colonization". We're generating stellar models already that appear fairly believable, and that wasn't terribly hard, but now we're trying to populate those stars with planets, planetoids and moons and things are getting exponentially more complex. While it would be killer to have computationally correct data about the orbital interactions, we just need "believable" metrics for planetesimal orbits. Thanks to earlier comments on Hill Spheres we're playing with random model−generation for moons now that look pretty accurate.
Our (known) universe is a pretty complex place, not the easiest thing to be modeling. ;) But we're trying, using the best possible data at hand. Thanks again for your input!

− Timothy Partee
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