Re: Epistemology 201: The Science of Science

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From: Jason (*jasonstevensNOSPAM_at_free.net.nz*)
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> >> >What makes maths different from any other random formal system is its
> >usefulness
> >> >in science.
>
> >> Mathematics isn't a formal system. I'll grant its usefulness to
> >> science. However, the attempted comparison with "any other random
> >> formal system" is bogus.
>
> >> > Science is about the world, therefore so is mathematics, by
> >proxy.
>
> >> That doesn't follow.
>
> >But maths is a formal system, or at least there is a formal system of
> >mathematics. That is, a formal grammar that describes the language of
> >mathematics, and rules of inference that describe legal moves from one
> >mathematical statement to another. Rules of inference from the empty string
are
> >the axioms of maths. You are correct in that it has not always this way, but
at
> >present, formal system theory is well established. This may change too of
> >course.
>
> You are perhaps referring to First Order Predicate Calculus (FOPC).
> And indeed, mathematicians do use FOPC. However, mathematics is not
> FOPC, and FOPC is not sufficiently expressible to allow it to be used
> exclusively.
>
> Given a particular system of axioms, say PA (the Peano Axioms),
> mathematicians could in principle use FOPC applied to those axioms.
> But mathematics is not confined to working within a particular axiom
> system. Moreover, the discussion axiom system itself is part of
> mathematics.

Maths is an extension of FOPC, like PA. The ZFC axioms are conventionally used and assumed, as far as I am aware. If another system is used in maths then people need to know about it. The ZF system without the axiom of Choice for

example, can lead to the creation of two spheres out of one in topology.

The study of axioms don't take place in maths. It is meta−logic or meta−maths that deals with this. Godels theorem for example is a meta−mathematical proof.

> >There are the various properties of formal systems, but what makes maths special
> >to us is not so much these properties but what we use it for. What bridges the
> >gap between maths as a formal system and maths as useful to us, is the semantics
> >we give it, our interpretation of the formal system of maths. But this is not
> >part of the formal system itself, which is just syntax, so a comparison to other
> >random formal systems is justified.
>
> I agree that what is important is the semantics, and not just the
> syntax. But that already argues that mathematics is more than just a
> formal system.

The bridge isn't the bank. It is only "more" when we link maths to the world.

When we look at "5" or "+" on a page, there is certainly an understanding that goes beyond the fact that they are just symbols on a page. But this is because we've grown up with these symbols and are use to how they work. This has been captured and formalised for the time being, even if the axioms are not invoked very often. And from my understanding, these axioms are prescriptive, not just descriptive of the way people do maths.

> >Since mathematics has evolved along−side science and plays a large part in
> >describing and predicting how the world works, then as a formal system goes, it
> >seems to be on the money as far as capturing something about the world.
>
> That's your opinion. As a mathematician, I have a different
> opinion. I consider it important that mathematics is not about the
> world. Roughly speaking, mathematics is about what would happen if
> reality did not intrude. We discover a lot about reality by seeing
> how it differs from the mathematical ideal.

Fair enough. The formal system of maths is ripe for exploration. People study it divorsed from the world. But why spend so much time on maths and not some other formal system? I think because of the close link maths has with the world.