Re: Poles an Zeros

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- *Date*: Fri, 29 Feb 2008 08:27:24 −0800

On Thu, 28 Feb 2008 19:14:25 −0800 (PST), Rob <robd@xxxxxx> wrote:

> Hello,
>
> Could someone explain to me Poles and Zeros.
> What does a Pole do and what does a zero do and how they interact
> together.
> How does phase, gain, and delays relate to them?
>
> If a book or a link could be recommended along with a what math needs
> to be used. (maybe a course outline to work from)
>
> I am right now working on systems which I have an good idea on how
> they work but I don't seem to have the underlying understanding or how
> to calculate.
>
> Some fully worked out examples I can look at would also be helpful. I
> just seem to be missing parts. I have found information. But I am
> having some issues putting it all together. A number of fields all
> converge and I am not able to find a path and seem to be jumping
> around and not getting anywhere.
> The area I am interested in are digitally controlled power converters
> (DC/AC and DC/DC power supplies).
>
> Thank you,
> Rob

OK, quick simplified version:

If you have a box with an input and an output, with linear response, there exists a mathematical expression, a transfer function, that describes how it behaves. If you know the transfer function, then for any input signal you can predict the output.

There are many ways to express the transfer function, most of them mathematically messy. Engineers prefer the Laplace Transform, which expresses the transfer function as a polynomial, using the complex varible "S". You'll have to read up on the theory to understand what S really is.

But if you express a transfer function as a polynomial on S, and factor it out nicely, and sweep S, the polynomial has "zeroes" where the numerator hits zero, and "poles" where the denominator hits zero. With a little practice, one can eyeball the equation, spot the poles and zeroes, and guess the frequency response.

Take this circuit:

```
input−−−−−−−−−−−−−−R−−−−−−−−+−−−−−−−−output
|
|
|
C
|
|
|
ground
```

which is a simple "single−pole" resistor−capacitor lowpass filter. If R = 1 ohm and C = 1 farad, it has a pole at radian frequency w = 1, namely at 0.16 Hertz.

The transfer function is

1 / (S+1)

and

Output = Input * 1 / (S+1)

so has a pole at S = −1, sort of. (S is actually a complex variable... dig into the theory for gory details.)

You estimate the frequency response by pretending that S is the input frequency, in radians/second. Ignore the sign!

For very small S, very low frequencies, 1/(S+1) = 1, so frequency response is flat, unity gain. At high frequencies, S is big, so 1/(S+1) is almost the same as 1/S, so the output is dropping off inverse with frequency.

If you graph the frequency response, it will look close to...

|

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|
|
1 | _________________________
| \
| \
G | \
A | \
I | \
N | \
| \
| \
| \
| etc forever
|
|
|
|
|
|
|
|
|
_______________________________________________________________
radian freq 1
```

where the gain is 1.00 at low frequencies up until 0.16 Hz, where it starts to roll off as 1/f, namely at −6 dB per octave. Engineers casually say that this frequency plot "has a pole at omega = 1" because the Laplace polynomial really does.

(Because S is actually a complex number, the rolloff region has a 90 degree phase shift. A 90 degree phase lag is associated with a 6 dB/octave rolloff in simple networks like this one.)

Now if you add a small resistor in series with that cap, say 0.1 ohms, it adds a zero. That looks like...

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1 | _________________________
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```

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| \
| \
0.1| −−−−−−−−−−−−−−−−−−−−−−−−−−−
|
|
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|
|
|
|
|
|
_____________________________________________________________
1 10
```

where the second break is at w=10, namely 1.6 Hz. Phase is 0 in the flats and 90 lag on the slopey part.

I find it helpful to start with a rough, practical explanation of stuff like this before I hit the books for the formal stuff.

John

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