Re: Exception to the rule? (Tarski´s T−scheme)

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Paul Holbach wrote in message
<881c8779.0406230653.15b5483f@posting.google.com>...

>> However, let DT be the theory in the
>> language of arithmetic L_{Tr} extended with a primitive predicate Tr(x),
>> containing the restricted T−scheme
>>
>> Tr("phi")<−>phi,
>>
>> for all sentences phi not containing Tr.
>
>So "phi" cannot be eg "This sentence is not true".

Right. That's the simplest restriction which yields a consistent theory of truth. It is in effect what Tarski proposed in 1933. Notice that the sentence "This sentence is not true (in the object language)" can be constructed, but it is a sentence of the meta−language, and so the T−scheme does not apply to it. Actually, this sentence is then a truth of the meta−langauge (actually, it is provable in the formalized truth theory). Tarski's solution to the Liar paradox is thus a hierarchy solution, where each language in the hierarchy is properly stronger than any earlier one (in particular, the truth predicate for language L_n only exists in language L_{n+1} and is undefinable in L_n).
Many recent authors have argued against Tarski's hierarchy solution and have presented three main objections

(i) Kripke's: we might have intertwining truth claims (his example involves
Nixon and Dean), which cannot be located in the hierarchy.
(ii) Putnam's: on the Tarskian approach, we cannot talk coherently of the
whole hierarchy, but it seems that in English we can.
(iii) On Tarski's approach, there is no single univocal notion of truth (or
truth predicate): there is only "truth in L", where L is some particular
interpreted language.

>Paraconsistentists such as Graham Priest willingly bite the bullet,
>accepting both the unrestricted T−scheme and inconsistency. For him "L
><−> ~L", i.e. "L & ~L" is true.

Right. But they have to modify both logic and the theory of truth is a really peculiar way, by moving to a 3−valued logic, with truth values T ("only true"), F ("only false") and B ("both−true−and−false").
I don't accept the dialetheist approach at all. They claim to have a theory of truth which is semantically closed: in which the many−valued object language contains its own semantics. I dispute this, and I have recently published a short paper (in Analysis, "Can a Many−Valued Language Functionally Represent its own Semantics?", October 2003) pointing out that such a many−valued language L cannot contain its meta−language, since the semantic valuation mapping ||.|| : Sent(L) −> {T, F, B} is not definable in the language L, at least not if the language contains constants "T", "F" and "B" and proves them distinct.

>> Roughly, the restricted T−scheme is not self−applicative, and only
applies
>> to the formulas of the _object−language_, and intuitively the formula
Tr(x)
>> means "x is a true sentence of the object language".
>
>
>I see.
>Unless I´m prepared to sacrifice consistency on the altar of
>dialetheism, I simply have to use the restricted T−scheme, haven´t I?

Well, not necessarily. Tarski's restriction is very severe and can be weakened to obtain (classically consistent) self−applicative truth theories. There are different restrictions, which allow self−application of the truth predicate. For example, Kripke's theory (Kripke 1975) is based on a three−valued logic with truth values {T, F, U}, where U is for "undefined", allows that phi and Tr("phi") always have the same truth value, even when phi contains the symbol Tr.
The corresponding deductive theory (formulated by Feferman) has the inference rules:

If you have proved phi, you can infer Tr("phi") Tr−intro
If you have proved Tr("phi"), you can infer phi Tr−elim

The formalized version of Kripke's theory (KF) also proves a scheme called T−Out.

(T−Out) Tr("phi") −> phi

even when phi contains the symbol Tr.

So, Tarski's severe restriction is not the only one available, although it is natural if you have a sharp distinction between object language and meta−language.

The Tarskian theory of truth over PA is sometimes called Tr(PA) or PA(S).
The Kripkean theory of truth over PA is called KF (Kripke−Feferman).
For details, see S. Feferman 1991, "Reflecting on Incompleteness", J. Symb. Logic.
There are other formalized theories of truth around. If you can read German, the best summary is a book by Volker Halbach 1996, _Axiomatische Wahrheitstheorien_ (Berlin).

>My initial point was that if it were the case that nothing exists,
>then there would be neither truths nor falsities, that is, neither
>true sentences nor false sentences.
>In other words, if nothing existed, something would be the case but
>nothing would be true!

Sort of. When we refer to "something being the case", we seem to be referring to a *proposition* (or *state of affairs*), not a sentence. What would be the case if there were nothing? The proposition that there is nothing would be the case. But would this proposition be *in* the world under consideration? Presumably not, since there is nothing there. This raises sticky questions about free logic, empty models, the nature of propositions/states of affairs, and so on.
But certainly you can't have the T−scheme, since it requires that at least two things exist.

−−− Jeff