

Re: Zorn's lemma for families of subsets of a countable set

Source: <http://sci.tech-archive.net/Archive/sci.logic/2004-06/1753.html>

From: Bill Taylor (w.taylor_at_math.canterbury.ac.nz)

Date: 06/24/04

Date: 23 Jun 2004 22:03:48 -0700

kramsay@aol.com (Keith Ramsay) wrote:

- > *Is the special case of Zorn's lemma where the ordered*
- > *set consists of a family of subsets of a countable set,*
- > *ordered by inclusion, a theorem of ZF?*

This doesn't quite parse properly, to me, though others seem to have had no trouble. I presume it means:–

There is a countable set, C , and we wish to well-order (some or all) subsets of it. Is it a ZF-theorem that this can always be done?

.....

The answer is obviously "no". WLOG, the countable set can be taken to be \mathbb{N} , (unless it's finite, making the result trivially true); then the set of ALL subsets of this is just \mathbb{R} , and cannot be well-ordered in ZF, as is well-known.

This seems so simple that I presume the original enquiry meant something else?

Oh hang on, I see it did – Zorn's lemma *starts off* with an ordered set of a particular sort, doesn't it. Damn. But I assume my argument will go through largely as is.

Bill Taylor W.Taylor@math.canterbury.ac.nz

Set theory is a shotgun marriage – between well-ordering and power-set. The two parties get along OK; but they hardly seem made for each other.
