Re: Humble pie.

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From: Andrew (*stan370_at_btinternet.com*)
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"Will Twentyman" <wtwentyman@read.my.sig> wrote

> >
> > There are two reasons for believing that the set of natural numbers can,
in
> > some sense, be 'complete'.
>
> You desperately need to define what you mean by complete. Anything
> relying on this concept is likely to be pure nonsense until it is defined.
>
> > The first depends upon Cantor's proofs of the
> > uncountability of certain sets. This is based upon the idea that for
any
> > supposed complete list of the elements of certain sets (such as the set
of
> > real numbers), a novel element not already in the list can be
demonstrated,
> > and after adding that to the list yet another novel element can be
shown,
> > and so on, and so on (e.g., via the diagonal argument). This is taken
as
> > proof that the set in question must have a higher order of infinity than
the
> > set of natural numbers since, even if supplied with a supposed complete
> > list, an infinity of elements guaranteed to not already be in the list
can
> > be demonstrated. If it can not be considered that the list is in fact
> > complete, then such demonstrations become meaningless, since it is no
great
> > achievement to show the existence of novel elements guaranteed to not
> > already be in the list if the list is known beforehand to be incomplete.
> > And that is the first reason for believing that the set of natural
numbers
> > can in some sense be 'complete'.
>
> You appear to be talking about whether a set is countable.
>

> > *The second reason springs directly from the first, in that, according to*
> > *such determinations, the set of natural numbers has an order of infinite*
> > *magnitude less than that of the set of real numbers because the real*
numbers
> > *can not be exhaustively paired, one−to−one, with any list of natural*
> > *numbers. Conversely, this means that the set of real numbers must be*
able
> > *to exhaust the set of natural numbers via one−to−one pairings (otherwise*
> > *one−to−one pairings have no meaning). In this sense, the set of natural*
> > *numbers must be considered to have been taken in its entirety, and so*
> > *actually be 'complete' in some sense.*
>
> *This makes very little sense. You appear to be talking about thing that*
> *are correct, but with very bizarre terminology.*

Ok, let's take things more slowly.

1) Speaking purely from the standpoint of cardinality, the natural numbers form an infinite set.
2) Again purely speaking from the standpoint of cardinality, the real numbers form an infinite set.
3) Cantor claims that the cardinalities of these two sets are different, namely aleph nought and aleph one respectively.
4) Cardinality relates to the quantity of elements in a set.

a) does this mean that the real numbers are uncountable? If not, why not?
b) if the real numbers are uncountable, does this mean that there are more individual elements in the set of real numbers than there are individual natural numbers? If not, why not?
c) if this does mean that the real numbers are uncountable, does this mean that a 1−1 pairing of the natural numbers with the real numbers will completely exhaust the set natural numbers whilst leaving some elements from the set of real numbers unpaired? If not why not?
d) if the natural numbers can be exhausted in this way, is it not legitimate to say that the set of natural numbers has been taken in its entirety − i.e., in a 'completed' form? If not, why not?

Andrew.