Possible Proof of the Lusin−Purves Theorem

Source: http://sci.tech−archive.net/Archive/sci.logic/2004−10/1215.html

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From: Acid Pooh (*poopdeville_at_gmail.com*)
Date: 10/27/04

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Date: 27 Oct 2004 04:28:18 −0700
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The Lusin−Purves theorem states that X,Y are standard Borel spaces and B is a Borel set in XxY such that each cross−section B_x is countable, then the projection of B onto X is a Borel set. I've seen several proofs of this fact, but I think I might have come up with a novel (though elementary) one. This sounds kind of crankish, especially since I don't seem to use the countability of each section in any significant way. Constructive criticism/counter−examples (but please, be gentle. I haven't slept in a few days)

Pf/ Since X and Y are Standard, XxY is Standard. We may assume that B is Borel since we can enlarge the topology on XxY so that B is closed and open. Thus B is Polish. The Cantor−Bendixson theorem implies that B can be written as B = O union P, where O is countable, P perfect, and O intersect P empty. Let a be any point in p and a_n any sequence in P converging to a. Since the projection mapping pi_X is continuous, if a_n −> a, pi_X(a_n) −> pi_X(a). Since every point in P is a limit point, pi_X(P) is closed, hence Borel.

Consider the family of sets which satisfy the condition B_x intersect P is empty. Since each of these sets is disjoint from P and countable, it follows that it is contained in O (in fact, the union of such B_x's is O). Since O is countable, there are only countably many x's in X such that B_x intersect P is empty. Thus the projection pi_X({(x,y) in B: B_x intersect P is empty}) is Borel.

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Any help salvaging this would be appreciated.
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