Re: Possible Proof of the Lusin−Purves Theorem

Source: http://sci.tech−archive.net/Archive/sci.logic/2004−10/1231.html

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Date: 10/27/04

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Date: Wed, 27 Oct 2004 12:33:31 −0500
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Acid Pooh wrote:

> The Lusin−Purves theorem states that X,Y are standard Borel spaces and
> B is a Borel set in XxY such that each cross−section B_x is countable,
> then the projection of B onto X is a Borel set. I've seen several
> proofs of this fact, but I think I might have come up with a novel
> (though elementary) one. This sounds kind of crankish, especially
> since I don't seem to use the countability of each section in any
> significant way. Constructive criticism/counter−examples (but please,
> be gentle. I haven't slept in a few days)
>
> Pf/ Since X and Y are Standard, XxY is Standard. We may assume that
> B is Borel since we can enlarge the topology on XxY so that B is
> closed and open. Thus B is Polish. The Cantor−Bendixson theorem
> implies that B can be written as B = O union P, where O is countable,
> P perfect, and O intersect P empty. Let a be any point in p and a_n
> any sequence in P converging to a. Since the projection mapping pi_X
> is continuous, if a_n −> a, pi_X(a_n) −> pi_X(a). Since every point
> in P is a limit point, pi_X(P) is closed, hence Borel.

I don't quite follow how this is supposed to show that pi_X[P]
is closed. Suppose x_0, x_1,... are elements of pi_X[P] converging
to x in X. Then you can pick a_0, a_1,... such that x_n = pi_X[a_n],
but how do you know that the a_n converge? I don't even see why
they should converge in the original topology on XxY, much less on
the new strengthened topology.