Re: Tautologies Then and Now

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Date: Thu, 09 Dec 2004 11:38:06 GMT
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"Chris Menzel" <cmenzel@remove−this.tamu.edu> wrote in message
news:slrncr9hto.4ss.cmenzel@philebus.tamu.edu...
> On Mon, 06 Dec 2004 19:43:18 GMT, robert j. kolker <nowhere@nowhere.com>
> said:
>> George Cox wrote:
>>
>>> For me (am I alone?) a tautology (in the logical sense) is a formula of
>>> propositional calculus which is true for all values of the truth values
>>> of its constituent atomic letters.
>>
>> Theorems of first order logic are also tautologies.
>

SH: Not the theorems, it seems like.

> In pretty much any logic text in existence, a tautology is a sentence in
> the language of propositional logic that is true regardless of the
> assignment of truth values to its atomic components. "Tautology" used
> in any other way, in the context of mathematical logic, is, well, wrong.
> The more general notion that covers both propositional logic and
> first−order (and higher−order) logic is that of a logical truth, i.e., a
> sentence of a given language that is true in all interpretations of the
> language. So, alternatively, a tautology is a logical truth of
> propositional logic.
>
> Chris Menzel
>

Would you comment on these quotes? * is my emphasis.. Particularly

"Because it is comprised of truth functional sentence schemata, a proxy may be tested for validity by the short−cut method of truth value assignment, or by *means of a truth table.*"

Paul wrote:
>>> Can you cite a text that extends truth tables beyond propositional
>>> logic? My professors always said that doesn't happen, and it certainly

>>> didn't in any of my texts.

http://www.lawrence.edu/fast/boardmaw/analytic_essay.html
"In Sentential Logic, we can prove an argument schema to be invalid by specifying a set of truth assignments to the sentential letters which results in true premises and a false conclusion; we thereby show that one line of the argument schema's truth table allows an interpretation having true premises and a false conclusion. In Predicate Logic, an argument schema typically consists of sentence schemata which are not truth functional: quantifiers, not truth functional connectives, are the major operators of the typical "quantified argument schemata."

*And quantifiers are not truth functional operators
since they may represent an infinite number of individuals; the truth value of a quantified sentence schema is therefore not a function of the truth values of any _finite_ number of simple sentence schemata.

*Nevertheless, we can test the validity of a quantified argument schema _indirectly_ by constructing and testing its _truth functional proxy for some_ (non−empty) _domain_ of a specified (finite) number of individuals; each of the premises, and the conclusion, in the original schema will be equivalent _in that domain_ to its truth functional counterpart in the proxy.

Because it is comprised of truth functional sentence schemata, a proxy may be tested for validity by the short−cut method of truth value assignment, or by *means of a truth table.*

And if a proxy proves to be invalid, it will provide a "recipe" for constructing an interpretation of the corresponding quantified argument schema into the same domain which will serve as a counter example, or refutation, to that argument schema. Thus, if the original quantified argument schema is valid, then _all_ of its corresponding proxies must also be valid. If _any one_ of the proxies corresponding to a quantified argument schema is invalid, then since it is therefore possible for the schema to have an interpretation into some domain under which its premises are true while its conclusion is false, the schema itself is invalid. Note that even though one particular corresponding proxy is valid, the original quantified argument schema might nevertheless be invalid: to be valid, _every_ corresponding proxy (for _every_ non−empty domain) must be valid."

SH: I quoted Peter Suber because your fame has not preceded you to my limited knowledge of who is a quotable authority in logic.

Regards,
Stephen