

sci.logic: Re: THIS STATEMENT HAS NO PROOF IN ANY SYSTEM = true or false?

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Now we're getting somewhere.

In article <1106604071.297554.238780@f14g2000cwb.googlegroups.com>,

<poopdeville@gmail.com> wrote:

>*I have no quarrel with Tarski's usage (though the redundancy theory of truth has been discredited for years). Your usage was slightly different, however. You \*asked\* if AC was true. Your usage and Tarski's are flatly incompatible.*

No, it isn't. I asked if AC was true. Following Tarski, this amounts to asking whether the cartesian product of nonempty sets is nonempty.

>*To wit -- the answer to the question*

>*"Is AC true?" is trivially "No" since we can construct sets in ZF for which it fails. I very much doubt this is what you want.*

The fact that there are models of ZF in which AC is false does not settle one way or the other whether the cartesian product of nonempty sets is nonempty.

For example, the sentence

If ZF is consistent then there are models of ZF in which AC is false

is provable in ZF (in fact in much weaker systems) and is consistent with both AC and its negation.

>*The sentence "the cartesian product of non-empty sets is non-empty," stripped of its context, is virtually meaningless.*

This is the key claim. Of course, things have to be taken in context.

The correct context for the sentence in question, however, is the class of all sets. It is not ZF or ZFC.

Let's take "ZFC is consistent" for comparison. It's true in some models of ZFC and false in other models of ZFC. Does that mean that it's

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meaningless unless I say whether I'm considering the "context" of a model of ZFC in which it's true or a model in which it's false? No. We \*know\* what it means for ZFC to be consistent, because we understand what symbols are, what strings are, what rules are, what integers are, and so forth. No infinite regress prevents us from grasping these things. Furthermore, just because we can exhibit models of ZFC in which "ZFC is consistent" can go either way doesn't somehow cause us to lose our ability to understand what we meant by the statement in the first place. And we can continue to ask, "Is ZFC consistent?" (equivalently, is "ZFC is consistent" true?) and know what we're asking without having to specify any model. In fact, models are irrelevant. The correct context for understanding "ZFC is consistent" is the world of arithmetic, or of syntax (arithmetic and syntax are mutually interpretable).

Similarly, the fact that AC is true in some models of ZF and false in others does not imply that it is meaningless for us to ask whether the cartesian product of nonempty sets is nonempty, in the context of the class of all sets.

Now, you might be skeptical about sets. In fact, it's pretty clear from your discussion that you \*do\* direct a kind of skepticism towards (infinite?) sets that you don't direct towards syntactic entities. I don't actually have any problem with your choosing to be skeptical of sets. What I do object to is the unfounded argument that independence results, or theories of truth (absolute vs. relative) have anything to do with such skepticism. Whether you prefer Tarski or Wittgenstein is actually irrelevant to the main issue at hand.

By the way, although this is mostly a philosophical debate, I would point out that I've noticed a rather strong correlation between getting these "philosophical" issues straight and getting simple mathematical facts straight. Recall my simple example above: the sentence

If ZF is consistent then there are models of ZF in which AC is false

is provable in ZF (in fact in much weaker systems) and is consistent with both AC and its negation. This is a straight mathematical fact, acceptable even to those who are skeptical about sets (but not of syntax). I suspect, though, that most people---perhaps 'cid' ooh included, though maybe I'm wrong about that---who claim not to understand what "the cartesian product of nonempty sets is nonempty" means will have some difficulty seeing this fact and its relevance to the discussion.

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The range of our projectiles---even ... the artillery---however great, will never exceed four of those miles of which as many thousand separate us from the center of the earth. ---Galileo, Dialogues Concerning Two New Sciences

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