Number of 2SATs in 3SAT

Source: http://sci.tech−archive.net/Archive/sci.logic/2005−02/1951.html

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Date: 02/25/05

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Date: Fri, 25 Feb 2005 14:57:44 -0800
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Number of 2SATs in 3SAT

Let V be a set of Boolean variables
and let F be a set of 3−clauses of
variables in V.

Let X be a subset of V such that
every clause in F contains at
least one variable in X.
Let x = |X| be the size of X.

If a variable in a 3−clause is given
an assignment then either the 3−clause
is satisfied or the 3−clause can be
reduced to a 2−clause.

F can be reduced to 2SAT by giving
an assignment to all the variables in X.
There are 2^x different 2SAT instances
that F can be reduced into.
If any of these 2SAT instances
has a solution then F is solvable, else
F has no solution.

Define x to be that smallest number of
variables one can randomly choose from V
to put in X and be sure every clause in F
contains a member of X.

There is a simple proof that x <= (n−2).
(where n is the number of variables in V)

Assume we have chosen a set of variables
to put in X such that there is exactly
one clause in F that doesn't contain
a variable in X.

This clause contains three variables not
in X and we only have to choose one variable
to satisfy this clause. This shows there
are at least two variables in V that don't
have to be in X.

x also depends on the number of clauses in F
and the clause to variable ratio.
I don't have a closed form formula for x, yet,
but x is fairly simple to calculate for
a given number of clauses and variables.
Hopefully, someone on sci.math can come up
with a closed formula for x.

Let V have n variables and F have m clauses.
Assume that no clause in F contains
the same variable more than once.

At least one variable in V must be in
at least 3*m/n clauses of F.

Let n=10 and m=40.

3*40/10 = 12 clauses removed by first variable
3*(40−12)/(10−1) =
3*28/9 = 10 clauses removed by second variable
3*18/8 = 7
3*11/7 = 5
3*6/6 = 3
3*3/5 = 2
3*1/4 = 1 clause removed by seventh variable

x=7 for n=10, m=40

The formula for x seems to be a little chaotic.
It looks like x depends on the clause to variable
ratio, be it also tends to grow as m grows.
For example, assuming n = m, x = n/2 for small
values of n and m, but x grows slowly for
larger values of n and m.

Russell
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