Re: Why an inconsistent ZF may be desirable, and should be welcome.

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From: Ross A. Finlayson (*raf_at_tiki−lounge.com*)
Date: 03/16/05

Date: 16 Mar 2005 15:03:48 −0800

Bhupinder Singh Anand wrote:
> *On Mar 15, 1:32 pm, Ross A. Finlayson wrote:*
>
> *RAF>> It seems then that you are not indicting ZF, but any theory.*
> *<<RAF*
>
> *Ross*
> ===
> *'Indicting'?! On the contrary, I am proposing that a set theory such*
as
> *ZF may be the appropriate language for the adequate expression of our*
> *mental concepts/constructs. Such a thesis could be of interest to the*
> *rapidly growing field of consciousness studies.*
>
> *It is not obvious to me why my argument of an inconsistency in ZF*
must
> *apply to any other theory. The inconsistency in ZF arises since the*
> *axiom of separation − due to Fraenkel − ensures that a primitive*
> *recursive function, and its arithmetical equivalent (cf. Theorem VII*
of
> *Goedel's seminal 1931 paper), are, both, represented in ZF by the*
same
> *set.*
>
> *The argument that I give shows that − although the two functions are*
> *equivalent in all their instantiations − we cannot always*
consistently
> *refer to their ranges as completed totalities − i.e., as well−defined*
> *mathematical objects. However, the axiom of separation ensures that,*
> *within ZF, the ranges of the two functions can always be treated as a*
> *common, completed, totality.*
>
> *Interestingly, ZF does distinguish the range of a primitive recursive*
> *function that, according to my argument − which is, itself, based on*
a
> *constructive/finitist interpretation of Goedel's reasoning − cannot*

be
> treated as a completed totality. It defines such a range as a
> recursively enumerable set that is not, however, recursive.
>
> As I argue elsewhere, such a, common, representation within ZF
obscures
> the possibility that, in some cases, the closure of a primitive
> recursive relation, and the corresponding closure of its arithmetical
> equivalent (cf. Goedel's Theorem VII), may not have the same meaning
> under their intuitive interpretation.
>
> RAF>> I have had what may be similar concerns ... <<RAF
>
> I'd be very interested in knowing more about your specific concerns,
> and their origins.
>
> RAF>> ... they have led me to a theory with no non−logical axioms,
> where everything resolves to a tautology, and the non−logical axioms
> that comprise ZFC besides regularity are considered theorems instead
of
> axioms. <<RAF
>
> Are you attempting to extend what Russell and Whitehead aimed at with
> their Principia, and suggesting that all mathematical languages might
> be interpretable in an appropriately formulated formal logic?
>
> RAF>> That leads to some elusion of the ramifications of some
> incompleteness results, which would apply to any theory with
> non−logical axioms. <<RAF
>
> Sorry, I've not grasped your intent here − could you amplify?
>
> RAF>> ... What's your background? <<RAF
>
> I am an independent scholar with an abiding (nearly 40−year old)
> interest in reviewing standard interpretations of Cantor's,
> Goedel's, Tarski's, and Turing's reasoning. In my essays,
> compiled only over the last four years, I address some grey areas in
> the foundations of mathematics, logic and computability. I believe
that
> non−resolution of some outstanding issues in these areas has
prevented
> − and continues to prevent − classical theory from developing
> mathematical languages of effective, and unambiguous, communication.
I
> argue that this limitation is, however, removable if we define
> mathematical objects formally, and mathematical truth verifiably.
Such
> definitions lead naturally to a finitist interpretation of
> meta−mathematics − in the sense of Hilbert and Bernays − and, also in
a

> *natural manner, address intuitionistic objections to some*
> *non−constructive, Platonic, elements that are implicit in standard*
> *interpretations of classical theory.*
>
> *Cf. http://alixcomsi.com/index01.htm*
>
> *Thanks for your kind, and encouraging, remarks, and for your interest*
> *in commenting on my posts at length.*
>
> *Regards,*
>
> *Bhup*

Hi Bhupinder,

Oh, I had misunderstood your point about the primitive and composite recursive functions having the same representation.

If they have the same representation I think they're the same thing.

I'm not familiar with the goals of Russell and Whitehead's Principia Mathematica, I think you are alluding to Goedel's two incompleteness results.

In addressing Goedel's incompleteness results, vis−a−vis the completeness result which I think leads to the contradiction you address about the functions with the same representation having different ranges due to completeness, basically I got to thinking that the Goedel sentence has an infinitely long chain of supporting sentences, thus that via induction there is a support for each, or that there are none and self−referential statements are validated by themselves representing true statements about the objects in the realm of discourse via tautology and the free propositional calculus, binary truth tables over predicates, in being self−referential eternally via self−referential induction.

I think NBG, for von Neumann−Bernays−Goedel, implicitly infinitely axiomatized ZFC, is to ZFC as hyperreal numbers are to real numbers.

My background is that around six years ago I learned about transfinite cardinality and have been arguing about it since. Then I got into the theory of numbers, addressing Goedel, about whose theories I still have much to learn, and philosophical foundations of mathematics, viz Finlayson numbers, number theory, numerical model, set theory, mathematics, ... Some progress has been made, I'm basically transparent in my arguments via postings to unmoderated discussion groups. Amateurs do it for love. I'm cavalier in my use of terminology.

So, I think to build a theory from an ur−element which is at once minimal and maximal, excluded middle, and tautology. I drone the

mantra "set of all sets, class of all classes", towards illumination of a construction of a theory that can be consistent, complete, and concrete, in a Platonist, constructivist, and pseudo−intuitionist sense.

Anyways my controversial points are not always agreed, many do gain ground over time. I'm quick to use sharp words without much training in their use.

I am not very familiar with the concepts of "recursive enumerability", where "recursively enumerable" is commonly abbreviated as "r.e.". Thus, I may be yet inadequately addressing your points, as a sincere and serious dilettante.

I enjoy reading your posts because you're sincere and well−informed, and seem willing to consider non−conventional viewpoints, which is necessary for progress. While that is so, if you say ZF is inconsistent the immediate conclusion of that statement is that ZF is inconsistent.

Warm regards,

Ross F.