Re: A little knowledge is a dangerous thing − THE HALTING PROOF

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On May 5, 9:15 pm, Charlie−Boo wrote in sci.logic:

CB>> 1. There is a way ("effective") that is not an algorithm. ... 2. There is a different way for each number, but no single way for all numbers. ... But maybe he can explain a little better than I can with my guesses. <<CB

Charlie
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Thanks for clarifying the issue with some incisive comments and analysis.

In footnote 36 of 'Some consequences of defining mathematical objects constructively and mathematical truth effectively', I clarify that:

"In other words, we argue that not every effective method is necessarily algorithmic, although every algorithm is an effective method. The possibility that "truth" may be non−algorithmic, and yet constructive, is implicit in Gödel's famous 1951 Gibbs lecture [Go51], where he remarks:

'I wish to point out that one may conjecture the truth of a universal proposition (for example, that I shall be able to verify a certain property for any integer given to me) and at the same time conjecture that no general proof for this fact exists. It is easy to imagine situations in which both these conjectures would be very well founded. For the first half of it, this would, for example, be the case if the proposition in question were some equation F(n) = G(n) of two number−theoretical functions which could be verified up to very great numbers n.' "

In footnote 98 I observe, further, that such a possibility is also implicit in Turing's remarks ([Tu36], §9, para II):

"Let P be a sequence whose n−th figure is 1 or 0 according as n is or is not satisfactory. It is an immediate consequence of the theorem of §8 that P is not computable. It is (so far as we know at present) possible that any assigned number of figures of P can be calculated, but not by a uniform process. When sufficiently many figures of P have been calculated, an essentially new method is necessary in order to obtain more figures".

Now, in his seminal 1931 paper, Goedel constructs an arithmetical relation R(x), and gives a constructive, and intuitionistically unobjectionable, meta−proof that the formula [R(n)] is provable for any given numeral [n] in any formal Peano Arithmetic, and that the formula [(Ax)(R(x)] is not provable in the Arithmetic.

Prima facie, using Occam's razor, a straightforward interpretation of this is that the arithmetical relation R(x) is effectively verifiable instantiationally, but that it is not uniformly (i.e., algorithmically) verifiable − hence it is not Turing−computable.

Such an interpretation would, of course, need to appeal to a provability thesis such as:

Provability Thesis (PT): A true arithmetic relation is Turing−computable, when treated as a Boolean function, if, and only if, it is PA−provable.

As a consequence of Turing's Halting Theorem, the thesis is essentially unverifiable. We are thus at liberty to either adopt it or to deny it.

Although classical theory does not appear explicitly committed to either case, standard interpretations of classical theory − including Goedel's own (implicitly Platonic, and arguably non−constructive) interpretations of his formal reasoning − do seem to conflict with the thesis.

In my above paper, however, I consider a constructive interpretation of classical theory in which PT follows as a consequence of some more fundamental theses.

Consequently, Goedel's reasoning would interpret as a constructive proof that R(x) is effectively true instantiationally, but that it is not Turing−computable.

It could, thus, be described as an instance of an effective method that ensures the effective verifiability of the truth of a denumerable set of arithmetical propositions, without recourse to an algorithm.

Regards,

Bhup

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