Re: .999... = 1

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- *From*: "Ken Quirici" <kquirici@xxxxxxxxx>
- *Date*: 29 Aug 2005 07:54:02 −0700

David C. Ullrich wrote:
> On 28 Aug 2005 21:37:41 −0700, "MoeBlee" <jazzmobe@xxxxxxxxxxx> wrote:
>
> >.999... = 1. As famous as this theorem is, I haven't been able to find
> >a rigorous proof (one that does not use infinite strings in additive
> >columns as if the strings were finite) and I haven't been able to prove
> >it for myself. I'd like to have a proof that:
> >
> >lim of SUM[k = 1 to n]9/(10^k) = 1.
>
> Assuming "lim" means the limit as n −> infinity then yes,
> that's exactly what "0.999... = 1" means, so it's exactly
> what you want to prove.
>
> >I tried the following (since this is such a familiar subject, such
> >things as that 'e' ranges over reals and 'n' over naturals are
> >implicit):
> >
> >Let f(n) = SUM[k = 1 to n]9/(10^k)
> >
> >Show that, for all e, there exists n, such that, for all k > n, |1 − e|
> >> |1 − f(k)|.
>
> But this is not the definition of limit. You need to prove this:
>
> (*) For any e > 0 there exists n such that for all k > n
>
> |1 − f(k)| < e.
>
> Note there are two differences there, regarding what e is and
> what inequality is required.
>
> Now how do you prove (*)? There are two ingredients:
>
> (i) f(k) = 10^(−k).
>
> (ii) For any e > 0 there exists a positive integer n such that
>
> 10^(−n) < e.

>
> From what you've written here it seems clear that you should
> have no trouble seeing how (i) and (ii) imply (*); the
> problem with your attempt at a proof was you had the
> definition of "limit" wrong.
>
> And you also should have no trouble giving a proof of (i)
> by induction. What about (ii)?
>
> In many expositions (ii) is just skipped over as obvious.
> But (ii) does require proof − for example there exist
> "ordered fields" (_roughly_, structures where _algebra_
> works exactly the same as it does for the real numbers)
> where (ii) is _false_!
>
> What's needed to prove (ii) is the "completeness axiom".
> It takes a minute to say what that is:
>
> Say S is a set of real numbers and b is a real number.
> We say b is a lower bound for S if b <= x for every x
> in S. We say S is bounded below if there exists a
> lower bound for S. The completeness axiom is this:
>
> Axiom: If S is a (nonempty) set of reals and S is
> bounded below then S has a greatest lower bound.
>
> Here "greatest lower bound" means exactly what it
> says: a lower bound which is greater than every
> other lower bound.
>
> For the rest of this post let
>
> S = {10^(−n) : n is a posiitive integer}.
>
> Then S is bounded below, for example 0 is a lower
> bound for S. So S has a greatest lower bound.
> In fact:
>
> (iii) The greatest lower bound of S is 0.
>
> If we know (iii) then (ii) follows: If e > 0
> and 0 is the greatest lower bound for S then
> e is not a lower bound for S, and (ii) says
> exactly "if e > 0 then e is not a lower bound for S".
>
> To prove (iii): We know that 0 is a lower bound
> for S. Suppose that 0 is not the greatest lower
> bound for S. Then there exists b > 0 such that
> b is a lower bound for S. But it's easy to see
> that if b is a lower bound for S then 10b is
> also a lower bound for S, and if b > 0 then

> 10b > b.
>
> So: If S has a strictly positive lower bound
> then S cannot have a greatest lower bound
> (contradiction), because given any positive
> lower bound b we can find a larger one, namely 10b.
> So S does not have a strictly positive lower
> bound, which says exactly that 0 is the
> greatest lower bound for S.
>
> QED.
>
> >Suppose |1 − e| > 1/(10^j) for some j > 0. Then let n = j. Let k > n.
> >Show, by induction on k, that 1/(10^j) > |1 − f(k)|.
> >
> >For the basis step, k = 0, the result is vacuously true, since k > n >
> >0. Or, the basis step could have k = 2 (since k > n > 0) and, in this
> >case, the basis step holds since if k = 2, then j = 1, and 99/100 is
> >closer to 1 than is 9/10. But at the inductive step I just get tangled
> >in calculations that don't seem to be leading to the result, as well as
> >I do not see how to use the crucial fact that 9 is the numerator. Maybe
> >I'm approaching this wrong from the start.
> >
> >This is not an exercise, as I just would like to have this proof as a
> >matter of record for a conversation about the subject. So, while hints
> >are appreciated, a finished proof would be even more appreciated. I
> >hope the proof doesn't require much more than the little I know, but if
> >the proof requires some more advanced theorems, then so be it, as I'll
> >need to learn these theorems anyway.
> >
> >Thanks in advance for any help you would provide.
> >
> >MoeBlee
> >
> >P.S. This is posted to sci.logic rather than sci.math, since I just
> >happen to be familiar with the postings of enough people in sci.logic
> >that I know there are some who I can trust their expertise.
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>
> David C. Ullrich

I'm confused about this (and vaguely disturbed).

Given that any irrational can be represented by a sequence of
rationals that approach arbitrarily close, and similarly, that any
rational can be approached by a sequence of rationals that approach
arbitrarily close (both by let's say Cauchy sequences − which
I take to be a particular way of 'approaching arbitrarily close'),
it would seem to me that

..99999....

represents, not 1, it's 'limit', but rather a particular Cauchy sequence of values approaching 1.

The notion of setting the sequence equal to its limit makes sense because for all practical calculations to which the sequence would be put in mathematics, the results would be identical if you used the limit rather than created infinite sequences of calculations tending to the result obtained just by going directly to 1.

I.e., mathematics would be hopelessly bogged down if you didn't replace any Cauchy sequence approaching a limit by that limit and consider yourself done.

However, to me, .99999... represents a sequence that tends to 1, rather than 1.

If you want to calculate using the limit of the sequence, use 1.
If you want to investigate the sequence, use the sequence, not 1. It may be that in some investigations the sequence is as important as the number it tends to.

Hope this makes some sense. I can't wrap my mind around the notation .99999.... = 1. It's apples and oranges, at least to my benighted mathematical logic.

Thanks.

Ken

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- ***Follow−Ups***:
  - ***Re: .999... = 1***
    - *From:* David C . Ullrich
  - ***Re: .999... = 1***
    - *From:* Robert Low
- ***References***:
  - ***.999... = 1***
    - *From:* MoeBlee
  - ***Re: .999... = 1***
    - *From:* David C . Ullrich
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