

## Re: Constructive Math query.

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- *From:* [stevendaryl3016@xxxxxxxxxx](mailto:stevendaryl3016@xxxxxxxxxx) (Daryl McCullough)
  - *Date:* 31 Aug 2005 05:51:54 -0700
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Bill Taylor says...

>\*IF\* one's brand of constructivism is of the hard-nosed type  
>that conflates truth with provability, (which seems to have  
>been widespread in the past, recall my excerpt from Bishop),  
>then I would have said it must be so.

I didn't think that constructivism conflates truth and provability.  
I missed your quote from Bishop, but Bishop's constructivism  
is *\*conservative\** over classical logic; anything that's constructively  
provable is also classically provable. The claim that

A is provable  $\leftrightarrow$  A

is certainly not true classically. Of course, you can  
take a faith-based approach to believe that if something  
is true *\*eventually\** a proof will be found...

>> Take an example in which A is neither provable nor refutable.  
>> The interpretation "If we can prove A, then B" becomes vacuously  
>> true, in that case, but "If A then B" isn't vacuously true.  
>  
>Well I would have thought that to a constructivist  
>(of this hard-nosed type) it WAS vacuously true.

As I said, I don't think that's true. Constructive logic  
doesn't prove anything that isn't also true classically.

I have another objection to the equivalence: If A and B  
are statements in the language of arithmetic, then

"If A then B"

is a statement about *\*natural numbers\**. In contrast,

"If we have a proof of A, then B"

is a statement about *\*proofs\**. They certainly don't *\*mean\** the  
same thing, even if they are both provably equivalent.

Re: Constructive Math query.

>Such a type would surely regard the "If A" part of your conditional  
>as equivalent to "If we can prove A" which he would deem false,  
>by noting the undecidability of A. So he would take your  
>conditional as vacuously true also. I see now that he would  
>also be required to admit that both A was false and  $\sim A$  was false.  
>This would presumably not trouble him as asserting  $\sim\sim A$  would  
>not require him to assert A.

If  $\sim A$  and  $\sim\sim A$  are both provable, that's as much of a contradiction as A and  $\sim A$ .

>Obviously I'm missing something trivial, but I doubt I'll ever  
>get it, I just can't seem to cotton to this philosophical  
>constructivism, (as opposed to the strictly formal approach  
>of Bridges for example).

Well, the way that makes the most sense to me is to think of propositions as corresponding to types (in the sense of computer science), and to think of a proof of a proposition as elements of that type. So for instance, the type of "A or B" is the disjoint union of the type of A and the type of B. The type of "A implies B" is the type of functions from the type of A to the type of B. The type of a falsehood is the empty type.

The philosophical issue, then, is this: Do there exist elements of a type *\*other\** than those that are constructively proved to exist?

On this question, I think of constructivists as *\*agnostic\** rather than *\*atheist\**. To take the atheistic position that nothing exists other than things that are constructively proved to exist seems contradictory in a way: There is no *\*proof\** that only constructive objects exist, and a constructivist should never assume the truth of something without proof.

Taking the agnostic position allows constructive logic to be applied even in a nonconstructive domain such as set theory. You can work out what are the constructive *\*consequences\** of nonconstructive *\*axioms\**. In the type-theoretic interpretation, axioms correspond to types containing elements that are not necessarily constructively definable.

—

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Re: Constructive Math query.

- **References:**

- ◆ **Constructive Math query.**

- ◆ *From:* Bill Taylor

- Prev by Date: **Constructive Math query.**

- Next by Date: **Re: An instance of Russell's paradox?**

- Previous by thread: **Constructive Math query.**

- Next by thread: **Question about arithmetical hierarchy notation**

- Index(es):

- ◆ **Date**

- ◆ **Thread**