Re: Moore on Skolem's Paradox

Source: http://sci.tech−archive.net/Archive/sci.logic/2005−10/msg00019.html

- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxx>
- *Date*: Sat, 01 Oct 2005 06:40:35 −0500

On 30 Sep 2005 11:30:33 −0700, "William of Ockham"
<d3uckner@xxxxxxxxxxxxxx> wrote:

>
>David C. Ullrich wrote:
>> On 29 Sep 2005 11:26:04 −0700, "William of Ockham"
>> <d3uckner@xxxxxxxxxxxxxx> wrote:
> 42. David C. Ullrich Sep 30, 11:36 am show options
>
>
>> If you're not sure what some button in some program
>> does the best thing is to just push the button and find out.
>> Aargh.
>
>What's wrong with this? This is my standard approach to software.

That doesn't surprise me one bit.

>Ockham:
>> An interpretation (as
>>I understand) is the information which we must have about a sentence in
>>order to understand it.
>
>Ullrich:
>> That's one of
>> the reasons I say you simply don't understand what you're talking
>> about − when we say things like "an interpretation of set theory"
>> that is simply _not_ what the word "interpretation" means.
>
>I got this from a textbook on model theory. It says "Sometimes we
>write or speak a sentence S that expresses nothing either true or
>false, because some crucial information is missing about what the words
>mean. If we go on to add this information, so that S comes to express a
>true or false statement, we are said to interpret S, and the added
>information is called an interpretation of S." The last bit says "the
>added information is called an interpretation of S".

That comes from a textbook on model theory?

Was it in the official technical part (the part you haven't
read yet), or just in some introductory "this is more or less
why we're about to do what we're about to do" section?

>Probably the textbook is wrong, but it makes me cautious of your
>exhortation to read textbooks.

Nobody exhorted you to read textbooks. People have said you should
read a good book on logic. I have no way of telling whether the
book that you quoted here is a good book on logic or not. If it
actually _is_ a _text_ on _model theory_ then you have not read it
yet, you've just skimmed through it, looking for the parts you
could understand. Which are unfortunately not the precisely
correct parts.

Anyway, I'm curious: what book is this?

Let me try to explain what's wrong with your assertion
that

'What we call "learning a
language" is assigning a reference to all of the terms that we learn.
So we learn that "hamburgers" mean hamburgers, "Socrates" means
Socrates, "two"means the number two, and so on. An interpretation (as
I understand) is the information which we must have about a sentence
in order to understand it.'

What you say here is true regarding one notion of "interpretation".
But that's not the notion of "interpretation" that's relevant to
things like "interpretation of set theory" in mathematical logic.
The two are related.

(a) Consider the sentence "2+2=4". That is a true sentence.
In order to know what it means we need to know what the
terms refer to − yes, when we learn English and then
when we learn arithmetic we learn what "2" means, what
"+" means, etc. Yes, learning those interpretations of
those symbols is in some sense part of learning the
language.

(b) Now consider the proposition "x+2=4". This is neither
true nor false in itself, it is true in some interpretations
and false in others:

(i) In an interpretation in which x = 2, "x+2=4" is true.
(ii) In an interpretation in which x = 3, "x+2=4" is false.

That's a (rough, informal, imprecise!) example of the notion
of "interpretation" that's relevant here. With this notion
of "interpretation" the things you've said about interpretations
are clearly false, unless you're suggesting that when we learn

the language we learn that x = 2, and we _also_ learn that x = 3.
If you're actually claiming that you're wrong − 2 is not what
"x" means, and neither is 3 what "x" means.

(One might say that when we learn the language of algebra
as expressed in English we learn that "x" means "a variable
which can take various values", or some such. Fine − that is
in some sense an "interpretation" of "x", which we do learn.
But that's an interpretation of "x" in a wholly irrelevant
notion of "interpretation".)

>On the use of the word "reference", my dictionary gives two relevant
>meanings: 1 the relation between a word and the object referred to, and
>2 the object referred to. This agrees with standard philosophical
>usage.

Fine. Using "referent" seems like a much better idea, because
it's more specific.

>Ullrich:
>> when you imagine a world where the proof of the existence of an
>> uncountable set is invalid you're just not making any
>> sense.
>
>I didn't say this. I said, suppose a world where, because of the
>meaning of the language people use, the reference of the expression
>"the real numbers" is different from what it actually is,

How can this have any bearing on anything? Yes, I can imagine
a world in which the phrase "real number" means "planet of
the solar system". In that world "the real numbers are uncountable"
is false (if "uncountable" has its usual meaning.) So what?

>so that, in
>their language, the proof of the uncountability of the reals is valid.
>
>
>> In that imaginary world you're speaking of, where everything
>> is countable, mathematics is simply nothing whatever like
>> mathematics in our world. If you want to imagine a world
>> where the sun is green and apples fall up, fine.
>[...]
>> Maybe what you have in mind is a world where the proof
>> is valid but nonetheless everything is "really"
>> countable. There is no such world.
>
>Perhaps not, but could there be? And why indeed couldn't we imagine
>such a world?

See (*) below.

> First, it would look exactly the same, given the fact
>that even a large but finite number of small things can appear
>continuous, as digital images prove.

If you actually think that aspects of the way we perceive
the physical world has any relevance here this is hopeless.
If you realize that it has no relevance but you're bringing
it up anyway this is hopeless. I give up.

> Second, our mathematics would
>seem exactly the same, and would even be true, so long as the model of
>our mathematical theory was one of the countable models we were talking
>about, and so long as any finite string that (in our world) defines a
>real number like pi, or root 2, has a counterpart in this imaginary
>world. Is it a physical impossibility? Why should it? Continuous
>motion requires the truth of analysis, but analysis is true in the
>imagined world. For example, the intermediate value theorem would be
>true for definable continuous functions (i.e. what we call "definable
>continuous functions").
>
>So, the inhabitants of this world apparently speak exactly the same
>language as us, but mean something slightly different by the phrase
>"for all x", which for us would mean "for all definable x", whatever
>sort of object x might be. In fact, they could even say that the reals
>were uncountable. What they'd mean by this in our terms is that there
>is no definable bijection between N and the definable reals, which
>there isn't because then they could apply a diagonal argument and
>define a real not in the image.
>
>Then why would mathematics be "nothing whatever like mathematics in our
>world."?
>
>What we cannot do, of course, is to imagine that this possible world is
>our world. For if so, there would be only countably many things, and
>set theory would be true, but set theory says there are uncountably
>many things. But then the inhabitants of this possible world would not
>be able to imagine this either. Indeed, they could even say to
>themselves, in their language "This possible world is not our world.
>For if so, there would be only be countably many things, and set theory
>would be true, but set theory says there are uncountably many things."
>And this would be true, because "there would be only be countably many
>things" expresses, in their language, the false idea that there would
>be no power set of the naturals. But "power set of X" refers, in their
>world, to all definable subsets of X! They simply cannot express what
>we, as external observers can express.
>
>On your argument that a relativity argument like this can "prove
>anything", not so.

[*]:

>The argument relies on the assumption that such a
>world is possible (i.e. the supposition of its existence would not
>result in contradiction)

>and that such a world would seem to its
>inhabitants exactly as our world seems to us. Ockham, Leibniz,
>Poincare and Einstein (and indeed Wittgenstein − the Private Language
>Argument is a relativity argument) all used relativity arguments to
>great effect.
>
>>Nothing about the learning process seems to rule this possibility out.
>> If the fixing of reference is
>>performed solely by subjective mental acts, then there is nothing to
>>prevent each of us fixing reference in a quite different way, which is
>>another way of saying that (in respect of our public language),
>>reference is not fixed at all.
>
>> This seems extremely silly, at least in the present context. The
>> fact that we can never be certain that what we mean by a term
>> is the same as what someone else means by the same term has
>> nothing to do specifically with the existence of uncountable
>> sets or Skolem's "paradox" − again, it's the sort of thing
>> you could use to refute anything anyone said about anything.
>
>You don't seem to understand the argument, which is that the fixing of
>reference cannot be performed solely by subjective mental acts, but
>must include some sort of "public" baptism.

You on the other hand don't seem to have very good reading
comprehension, if you think what you just said is a refutation
of what I said, as opposed to just another way of explaining
why it's just silly, on can−be−used−to−refute−anything grounds.

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David C. Ullrich
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- ***Follow−Ups***:
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  - ♦ ***Re: Moore on Skolem's Paradox***
    - ◊ *From:* Daryl McCullough

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