# What is the 1st order formal system known as PA?

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A 1st order formal system T is, of course, defined by:



i)   1st order language L(T),
ii)  logical axioms of L(T), plus some non−logical axioms,
iii) some rules of inferences.



[based on Shoenfield's "Mathematical Logic".]



So what would make a system T unique from another, say, T'?
Obviously either L(T) is different from L'(T'), or non−logical
axioms of T are different than those of T', or both. Suppose though
T and T' are of the same language L [i.e. L(T) = L(T')], then T and
T' could be different iff at there is at least one non−logical
axiom, say, A of T differs from one axiom A' of T' [assuming both
T and T' have the same number of axioms].



But when could 2 formulae A, A' of the same language become 2
different axioms? There seem to be only 2 cases:



(a) A and A' are syntactically different.
(b) A and A' are syntactically _identical_, but the n−ary relation(s)
    stipulated in this one formula are different, from one being's
    point of view to another's.



It's the case (b) that, imho, PA has an issue that has long been
neglected. The issue is that from the theory, or FOL framework,
level [and not necessarily from the model level], given any successor
function S of PA, which is 1−1, one always could infer another
_different_ successor function S' that one could use to define the
binaries + and *. Therefore, when 2 [human?] beings refer to PA
theory, they might in fact be talking about _2 different PA theories_.
Or, shall we say 2 different PA−ish theories! [Note that all PA−ish
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theories are of the same language L(PA)]

Two consequences seem to stem from such multi−PA−ish−theories observation:

(1) what we normally consider as a (PA) theorem now should be a _common_ theorem: a theorem in all the PA−ish theories.
(2) Given a particular formula F of L(PA), it's quite possible that one could not − within the framework of FOL − determine a particular PA−ish theory in which F is a decidable, or undecidable.
(3) It's suspected that: if GC (Goldbach Conjecture) is false then it's provable in all PA−ish theories; otherwise one could not use FOL framework to determine which of the PA−ish theories GC is (un)decidable.

Just an opinion though. And I'd like to thank in advance for any constructive correction, or comments.

−−−Nam

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The difference between landscape and landscape is small,
but there is a great difference between the beholders.
Ralph Waldo Emerson
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