Re: What is the 1st order formal system known as PA?

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- *From*: "MoeBlee" <jazzmobe@xxxxxxxxxxx>
- *Date*: 19 Nov 2005 18:28:36 −0800

Rupert wrote:
> Usually there is no predicate "is a number".

Then there's a fair amount of terminological slippage among writers. When I look up 'Peano arithmetic' or 'Peano axioms', even among contemporary sources, I usually get a list of five axioms that are pretty close to five of the nine axioms in Peano's paper (the other four are axioms about identity), as I cast this in first order logic with identity (I omit leading univesal quantifiers):

0 0−place function symbol
S 1−place function symbol
N 1−place predicate symbol

Axioms:

N0

Nn −> NSn

~(Nn & 0=Sn)

(Nn & Nk & Sn=Sk) −> n=k)

(phi[0] & An(Nn −> (phi[n] −> phi[Sn]))) −> An(Nn −> phi[n])

[I'm confused about Peano's own formulation as it is given on page 94 of van Heijenoort's 'From Frege To Godel'. There, axiom 7 does not seem to state that S is 1−1. Is this the result of typos? Should the second '=' be the symbol for material implication instead, and the antecedent and consequent reversed?]

But Shoenfield on page 204 of 'Mathematical Logic' specifies Peano arithmetic as axiomatized in first order logic with identity and omitting leading universal quantifiers (cf. page 22 also):

0 0−place function symbol
S 1−place function symbol

+ 2−place function symbol
* 2−place function symbol
< 2−place predicate symbol

Sn not= 0

Sn=Sk −> n=k

n+0=n

n+Sk=S(n+k)

n*0=0

n*Sk=(n*k)+n

n not< 0

n<Sk −> (n<k v n=k)

(phi[0] & An(phi(n) −> phi[Sn])) −> An phi[n]

So am I correct to take it that in current discussions, by 'Peano arithmetic', people mean the theory axiomatized by the above axioms? (It could be a different set of axioms as long as it axiomatizes the same theory.) Also, that each of those axioms is independent of the set of the others? But, as to independence of primitive symbols, couldn't we take '<' as defined by:

Df. n < k <−> Ej(j not= 0 & n+j=k)

> > Also, in set theory we prove that any two Peano systems are isomorphic.
> > And, if I'm not mistaken, we prove that any two completely ordered
> > fields are isomorphic. Yet, Lowenheim−Skolem tells us that for the
> > axioms of Peano arithmetic there are non−isomorphic models, and for the
> > axioms for a complete ordered field, there are non−isomorphic models.
>
> The axioms for a complete ordered field are second−order.
> Loewenheim−Skolem applies to first−order theories.

Yes, I overlooked that. To speak of bounds on subsets of the domain, we have to speak of sets, so that we're in set theory or we can cast the matter in second−order. But, to express subsets of the domain couldn't we use an axiom schema with phi, for formulas, instead of using sets or quantifying over second−order predicate symbols; thus we could express the axioms of a complete ordered field as a first order theory with axioms and one axiom schema?

And, by noting for myself that indeed 'Peano arithmetic' denotes different things depending on the writer or context, I understood the rest of your comments.

Thanks for you remarks. They have been helpful.

MoeBlee

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- ***References***:
  - ***What is the 1st order formal system known as PA?***
    - *From:* Nam Nguyen
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    - *From:* David C . Ullrich
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