

# Re: What is the 1st order formal system known as PA?

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*Source:* <http://sci.tech-archive.net/Archive/sci.logic/2005-11/msg00409.html>

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- *From:* "Rupert" <[rupertmccallum@xxxxxxxxxx](mailto:rupertmccallum@xxxxxxxxxx)>
  - *Date:* 19 Nov 2005 21:45:24 -0800
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MoeBlee wrote:

> Rupert wrote:

>

>> The answer to this, I suppose, is that when writers present the axioms  
>> informally they might throw around phrases like "is a number" which  
>> don't actually show up in the formalization. This is about all I can  
>> say unless you can point me to a specific writer.

>

> Perhaps not from a renowned writer, but, for example, here is what you  
> get as the very first hit on a Google search for 'Peano arithmetic'  
> (the second link is just the one suggested at the page of the first  
> link):

>

> <http://mathworld.wolfram.com/PeanoArithmetic.html>

>

> <http://mathworld.wolfram.com/PeanosAxioms.html>

>

This is an informal presentation of Peano's axioms, probably modelled on the presentation Peano himself originally gave. It's different to a formal presentation.

>>> But Shoenfield on page 204 of 'Mathematical Logic' specifies Peano  
>>> arithmetic as axiomatized in first order logic with identity and  
>>> omitting leading universal quantifiers (cf. page 22 also):

>>>

>>> 0 0-place function symbol

>>> S 1-place function symbol

>>> + 2-place function symbol

>>> \* 2-place function symbol

>>> < 2-place predicate symbol

>>>

>>>  $S_n \neq 0$

>>>

>>>  $S_n = S_k \rightarrow n = k$

>>>

>>>  $n + 0 = n$

Re: What is the 1st order formal system known as PA?

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>>>  $n+S_k=S(n+k)$

>>>

>>>  $n*0=0$

>>>

>>>  $n*S_k=(n*k)+n$

>>>

>>>  $n \text{ not} < 0$

>>>

>>>  $n < S_k \rightarrow (n < k \vee n=k)$

>>>

>>>  $(\text{phi}[0] \ \& \ \text{An}(\text{phi}(n) \rightarrow \text{phi}[S_n])) \rightarrow \text{An } \text{phi}[n]$

>>>

>>> So am I correct to take it that in current discussions, by 'Peano

>>> arithmet