Re: What is the 1st order formal system known as PA?

Source: http://sci.tech−archive.net/Archive/sci.logic/2005−11/msg00435.html

- *From*: "MoeBlee" <jazzmobe@xxxxxxxxxxx>
- *Date*: 20 Nov 2005 17:29:03 −0800

Rupert wrote:
> This is an informal presentation of Peano's axioms, probably modelled
> on the presentation Peano himself originally gave. It's different to a
> formal presentation.

It is much more than a difference in formality. The theory shown at the web site, which is in the same vein as Peano's own formulation is a much weaker and less expressive theory (for lack of '+' and '*', which I don't think I'm mistaken in thinking that they are not definable from just '0' and 'S' along the few axioms at that website) than what we've agreed is PA. And that web site is hardly an exception.

> > So Chang & Keisler don't include the seventh and eighth axioms of
> > Shoenfield. And I don't think those axioms are derivable, even with the
> > definition of '<', from the first six axioms along with axiom nine, are
> > they?
>
> Yes, they are.

Yep, you're right, and trivial too. Thanks. So Shoenfeld gives superfluous axioms. Hmm.

So here's what we agree is first order PA:

First order logic with identity and language:

0
S
+
*

Axioms:

(1) Sn not= 0

(2) Sn=Sk −> n=k

(3) n+0=n

(4) n+Sk=S(n+k)

(5) n*0=0

(6) n*Sk=(n*k)+n

(7) (phi[0] & An(phi(n) −> phi[Sn])) −> An phi[n]

> > Right, I don't know all the details of that, but I have a sense of the
> > outline. However, what I don't understand is that within set theory we
> > show that any two complete ordered fields are isomorphic with one
> > another. Yet, Lowenheim−Skolem tells us that there are non−isomorphic
> > models of the axioms,
>
> Which axioms?

(1) (x+y)+z = x+(y+z)

(2) x+y=y+x

(3) Ey x+y=z

(4) (x*y)*z = z(x*y)

(5) x*y=y*x

(6) x+y not= y −> Ez x*z=y

(7) x*(y+z) = (x*y)+(z*z)

(8) (x<y & y<z) −> x<z

(9) x not< x

(10) x<y v y<x v x=y

(11) x<y −> x+z<y+z

(12) Theorem: E!xAy y+x=y

(13) Definition: 0 = the x such that Ay y+x=y

(14) (x<y & 0<z) −> x*z<y*z

(15) Schema: (Ew phi[w] & ExAy(phi[y] −> (y<x v y=x))) −>
Ez(phi[z] & Ay(Ax(phi[y] −> (y<x v y=x)) −> y not< z))

If we were in set theory, so that all the axioms were relativized to a
certain sets S, and alternatively to a set S' and phi[x] were taken as

xe{y| yeS & phi[y]} and as xe{y| yeS' & phi[y]}, respectively, then we prove that S with its operations (+ on S, * on S) and ordering (< on S) is isomorphic to S' with its operations (+ on S', * on S') and ordering (< on S').

Yet, in the meta−theory, which is itself set theory (a copy, up a level, of the set theory in the object language?), there are non−isomorphic models of the axioms, even though, within set theory in the object language, any sets satisfying the conditions embodied in the axioms are isomorphic.

So, maybe I've just reiterated the "paradoxical" aspect of Lowenheim−Skolem? Have I just reiterated, in a long around way, that a set is evaluated differently from within set theory than it is in the metatheory?

> You seem to be assuming that there is a set of first−order axioms such
> that any model of them is a complete ordered field. This is not the
> case.

So what I need to say instead is that in set theory, any set that satisfies the axioms above is a complete ordered field if we modify them to be set−theoretical without the schema (as I described) but that not all models of the axioms in their original form, with the schema, are complete ordered fields, right?

Thanks,

MoeBlee

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- ***Follow−Ups***:
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* Rupert
- ***References***:
  - ***What is the 1st order formal system known as PA?***
    - *From:* Nam Nguyen
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* David C . Ullrich
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* MoeBlee
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