Re: What is the 1st order formal system known as PA?

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- *Date*: 20 Nov 2005 22:07:44 −0800

Rupert wrote:
> They're not superfluous if you're taking "<" as a primitive term. They
> are superfluous if you take it as a defined term.

But it's definable. I like the short list much better.

> I have my doubts about whether you really can prove the uniqueness of
> additive and multiplicative inverses from those axioms.

I'll try to remember to double check the arguments. They're in
Browder's 'Mathematical Analyis'.

> > If we were in set theory, so that all the axioms were relativized to a
> > certain sets S, and alternatively to a set S' and phi[x] were taken as
> > xe{y| yeS & phi[y]} and as xe{y| yeS' & phi[y]}, respectively, then we
> > prove that S with its operations (+ on S, * on S) and ordering (< on S)
> > is isomorphic to S' with its operations (+ on S', * on S') and ordering
> > (< on S').

> Not true. This would only be true if you replaced your first−order
> axiom schema by a second−order version of the least upper bound
> principle. As things stand, your axioms have nonisomorphic models.

Don't forget, I'm not talking about models for a language. I'm talking
about structures within set theory.

Here's the statement in Browder:

Theorem. There exists a complete ordered field R. If R1 and R2 are
complete ordered fields, there exists an isomorphism of ordered fields
between them, i.e., there exists a bijective mapping psi:R1−>R2 which
preserves the structure: for any x,yeR1 we have psi(x+y)=psi(x)+psi(y)
and psi(x*y)=psi(x)*psi(y); for any x,yeR1 with x<y, we have
psi(x)<psi(y).

He's not in formal set theory, but he might as well be since there's
nothing he's doing that can't be cast in set theory.

> > Yet, in the meta−theory, which is itself set theory (a copy, up a
> > level, of the set theory in the object language?), there are
> > non−isomorphic models of the axioms, even though, within set theory in
> > the object language, any sets satisfying the conditions embodied in the
> > axioms are isomorphic.
> >

> Hmmm. I thought when you said "if we were in set theory" you were
> proposing to take set theory as the metatheory. Now it looks like you
> were proposing to take set theory as the object theory. That's
> incoherent. The object theory is the first−order theory you were
> discussing. That's different to set theory. Only the metatheory can be
> set theory. And when we take set theory as the metatheory, as I said,
> we prove that your theory has nonisomorphic models. It's only the
> second−order version of the theory that has a unique model up to
> isomorphism.

What I mean is that set theory is a first order object level theory,
call it Z0. And the first order formal meta theory is set theory, call
it Z1. Z1 is just like Z0 but a level up. Z0 is first order set theory
at the object level and Z1 is first order set theory at the meta−level.
Why is that not allowed? What would be the formal meta theory for
object level set theory? Why can't it be formal meta level set theory?
It might as well be, since the informal meta theory is set theory.
Might as well formalize it. (This is not circular; but it is infinite
escalation, since there is Z2, Z3,....) But when I said "we're in set
theory", I meant the object level.

> > So, maybe I've just reiterated the "paradoxical" aspect of
> > Lowenheim−Skolem? Have I just reiterated, in a long around way, that a
> > set is evaluated differently from within set theory than it is in the
> > metatheory?

> I can't see how what you're saying is making any sense. There's an
> object theory, a first−order theory which you presented. For the
> metatheory we can take set theory. And then we prove that the object
> theory has non−isomorphic models. That's all there is to it. If for our
> object theory we had the usual second−order theory, we could prove that
> the object theory has a unique model up to isomorphism, but that's a
> different object theory. There's no contradiction.

Don't forget, I'm not using any second order language or second order
theory. I have first order object language theories, such as Z0
(Zermelo set theory), PA, axiom sets about fields, etc. Now, I see that
the so−called field axioms or so−called axioms for groups, etc. can
also be not axioms, but rather, definitions, in Z0. Then in Z0 we show
that any structure (set theory structure, like an algebraic structure;
not a structure for a language) that satisfies the definition is
isomorphic to any other structure that satisfies the definition. That
prove is in, for example, Browder.

But, reverting back from definitions, I also have a separate first
order object level theory, not set theory, but still a first order
theory, that uses the phi[x] style schema. My set theory definition of
a certain kind of structure is "translatable" back to an object
language theory in a different language (as primitives it has + * <,
rather than set theory which has only e, for 'is an element' while + *
< are defined) in which the definitions are not definitions but rather
axioms. Let's call this B. B is a first order object level theory with
the so−called field axioms and the least upper bound principle as an
axiom schema.

Also, I have a formal first order meta theory that is also a first
order set theory, Z1 (exactly like Z0, except "declared" to be up a
level. I.e., a formal first order meta theory, that is set theory, for
a formal first order object level theory that is set theory. In this
meta theory is where I assign structures for languages and have models.
So, in this meta theory, I study object theories, including Z0 and B
and PA. But I see that not all models of B are isomorphic, even though
all structures in object level Z0 that satisfy the definitions (just
like the axioms of B, but instead defintions in Z0) are isomorphic.

The situation is that the object level set theory structures are
isomorphic but the meta level models are not isomorphic. I'm guessing
that this is a reflection of Lowenheim−Skolem.

(One more thing: I take the domains of the models and the relations and
functions of the model to be sets in the meta theory set theory.)

> If you take the second−order version of the axioms, then every model is
> a complete ordered field, but regarding the first−order theory you were
> discussing, not every model is a complete ordered field.

I understand that. And what I'm saying is that that is parallel to the
situation in which all the structures are isomorphic in first order
object level set theory; but not all models are isomorphic in first
order meta level set theory. Because one is isomorphism of structures
in a theory, and the other is non−isomorphism among models that are
structures for first order languages. Just like, in first order object
level set theory, the set of reals is uncountable, but in first order
meta level model theory (expressible within first order meta level set
theory) there are countable models if there are infinite models.

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- ***Follow−Ups***:
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* Rupert

- ***References***:
  - ***What is the 1st order formal system known as PA?***
    - *From:* Nam Nguyen
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* David C . Ullrich
  - ***Re: What is the 1st order formal system known as PA?***
    - *From:* MoeBlee
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