Re: incompleteness of first−order logic

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Li Yi wrote:

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        1.
        Give a concrete counterexample to:
        For any theory T and sentence p,  T |/- p => T |- not p.
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Assume completeness, i.e. T |= A iff
T |- A. Then take for example the
void Theory T. It is neither T |- p
nor T |- ~p.

Because you can find M1={p} with

   M1 |= T and not M1 |= ~p

Hence it is not T |- p.

And M2={~p} with

   M2 |= T and not M2 |= p

Hence it is not T |- ~p.
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        2.
        M is a model. Let Th M = {p : M |= p}.
        Show that Th M is a complete theory, that is to say, Th M |/= p => Th M
        |= not p.
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This is not true. Th M = {p : M |= p} is
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not a complete theory.

Take for example M2 from above, then Th M2
is the void theory, and as was shown above,
the void theory is not complete.

Bye
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