Re: interpolation theorem of propositional logic

Source: http://sci.tech−archive.net/Archive/sci.logic/2006−04/msg00143.html

- *From*: Jan Burse <janburse@xxxxxxxxxxx>
- *Date*: Thu, 13 Apr 2006 13:24:15 +0200

David C. Ullrich wrote:

> On Wed, 12 Apr 2006 13:29:30 +0200, Jan Burse <janburse@xxxxxxxxxxx>
> wrote:
>
>> Hi
>>
>> David C. Ullrich wrote:
>>
>>> On 11 Apr 2006 03:36:16 −0700, "Li Yi"
>>> <liyi.cn@xxxxxxxxx> wrote:
>>>
>>>> If alpha |= beta, then there is some gamma
>>>> all of whose sentence
>>>> symbols occur in both alpha and beta and
>>>> such that alpha |= gamma |=
>>>> beta.
>>>
>>> This is obviously false.
>>> Hint: The weaker statement "If alpha |= beta, then there is
>>> some gamma all of whose sentence symbols occur in both
>>> alpha and beta"
>>> is obviously false.
>>
>> Depends on what one understands by sentence symbols.
>
> The subject line specifies _propositional_ logic.
> There's a perfectly standard notion of "sentence
> symbol" in propositional logic

> > If for example sentence symbols means variables, function
> > symbols and predicate symbols,

> and none of these exist in propositional logic.

If you restrict FOL to 0−ary predicate symbols,
even not allowing equality, you arrive a propositional
logic.

> These things do of course exist in predicate logic.
> Calling them "sentence symbols" seems like maximally
> strange terminology; the things that they "represent"
> are not sentences.

A propositional variable is a sentence symbol.
Because a propositional variable in essence can
represent a full propositional formula. This
can be done either by using biimplication, i.e.
for example:

p <−> q & ~r.

Now p stands for q & ~r. Or by explicit substitutional
rules and/or lemmas.

For example many natural deduction systems
come with the rule, that if A is an axiom,
the one can use A[S] where S is a substition
from propositional variables to propositional
formulas.

Also there are lifting lemmas, that say for
example if A is a tautology then A[S] is
also a tautology.

Etc..
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