Re: Godel's incompleteness theorem vs Church's/Turings work

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- *From*: "Peter_Smith" <ps218@xxxxxxxxx>
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george wrote:

> Frederick Williams wrote:
>
> > Incompleteness and undecidable are different things.
>
> No, they're not.
> In order for any theory−of−the−kind we're talking
> about to be incomplete, there must necessarily be
> statements in its language that it does not decide.

Take a toy case: a theory whose language consists of the propositional atoms P, Q, and R, plus the truth−functional connective, whose sole axiom is (say) P & Q, and whose logic is a standard natural deduction system for propositional logic.

This theory is (i) negation incomplete [it doesn't decide whether R or not−R, for example]. But (ii) it is a decidable theory −− i.e. the question is "X a theorem?" for any sentence X in its language is algorithmically decidable e.g. by a truth−table test on "(P & Q) −−> X".

Moral: the fact that there are statements in a theory's language that it doesn't decide does NOT entail that it is an undecidable theory.

> > This is illustrated by FOL:
>
> No, it isn't.
> FOL is a logic.
> PA and all the things Godel was talking about
> are THEORIES not NOT logics.

Both off−message here! (i) You can think of FOL as the theory whose logic is first−order and whose set of non−logical axioms is null. But

(ii) of course −− as was pointed out −− FOL (although complete in the sense of being having a deductive system that is complete with respect to the standard semantics) is not a negation−complete theory (it doesn't settle every sentence X in its language!).

> You earlier also characterized FOL as "undecidable". That is inappropriate. "Undecidable" does not reasonably DIRECTLY apply to anything as big as a theory, let alone something even BIgger like all of FOL. "Undecidable" is properly and directly applied to ONE INDIVIDUAL sentence.

Not so. It is entirely standard to talk of theories being undecidable. And appropriately so −− the issue is whether there exists an algorithm that applies *theory−wide* to detemine the theoremhood of arbitrary sentences.

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