Re: interpolation theorem of propositional logic

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Fri, 14 Apr 2006 05:28:26 −0500

On Thu, 13 Apr 2006 13:24:15 +0200, Jan Burse <janburse@xxxxxxxxxxx>
wrote:

> David C. Ullrich wrote:
>
> > On Wed, 12 Apr 2006 13:29:30 +0200, Jan Burse <janburse@xxxxxxxxxxx>
> > wrote:
> >
> > > Hi
> > >
> > > David C. Ullrich wrote:
> > >
> > > > On 11 Apr 2006 03:36:16 −0700, "Li Yi"
> > > > <liyi.cn@xxxxxxxxx> wrote:
> > > >
> > > > > If alpha |= beta, then there is
> > > > > some gamma all of whose
> > > > > sentence
> > > > > symbols occur in both alpha
> > > > > and beta and such that alpha
> > > > > |= gamma |=
> > > > > beta.
> > > >
> > > > This is obviously false.
> > > > Hint: The weaker statement "If alpha |= beta,
> > > > then there is some
> > > > gamma all of whose sentence symbols occur
> > > > in both alpha and beta"
> > > > is obviously false.

Depends on what one understands by sentence symbols.

The subject line specifies _propositional_ logic.
There's a perfectly standard notion of "sentence
symbol" in propositional logic

If for example sentence symbols means variables, function
symbols and predicate symbols,

and none of these exist in propositional logic.

If you restrict FOL to 0−ary predicate symbols,
even not allowing equality, you arrive a propositional
logic.

Supposing that's so, it's hard to see what your
point could be. Second, _if_ we restrict FOL as you
say, and _if_ we say that the resulting theory _is_
propositional logic (instead of saying more correctly
that it's isomorphic to propositional logic) then
variables and function symbols do not exist in
propositional logic.

First, and more to the point: _If_ we do that
we arrive at a situation where the statement
I made that you disputed is obviously correct:
Say P and Q are unary predicates. Let alpha be
P −> P and let beta be Q −> Q. Then
alpha |− beta, although there does not exist
a gamma including only sentence symbols common
to alpha and beta.

These things do of course exist in predicate logic.
Calling them "sentence symbols" seems like maximally
strange terminology; the things that they "represent"
are not sentences.

A propositional variable is a sentence symbol.

I wasn't disputing that. A propositional variable

is indeed a sentence symbol. It doesn't follow that
"variables, function symbols and predicate symbols"
are sentence symbols.

> Because a propositional variable in essence can
> represent a full propositional formula. This
> can be done either by using biimplication, i.e.
> for example:
>
> p <-> q & ~r.
>
> Now p stands for q & ~r. Or by explicit substitutional
> rules and/or lemmas.
>
> For example many natural deduction systems
> come with the rule, that if A is an axiom,
> the one can use A[S] where S is a substition
> from propositional variables to propositional
> formulas.
>
> Also there are lifting lemmas, that say for
> example if A is a tautology then A[S] is
> also a tautology.
>
> Etc..

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David C. Ullrich
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