Re: interpolation theorem of propositional logic

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxx>

- *Date*: Fri, 14 Apr 2006 05:32:17 −0500

On Thu, 13 Apr 2006 13:30:06 +0200, Jan Burse <janburse@xxxxxxxxxx>
wrote:

> Hi
>
> In FOL a prim formula can stand for a
> full formula. The prim formula then has
> so to speak parameters. For example:
>
> odd(X) <−> X mod 2=1
>
> But there are some problems with recursivity,
> already in propositional logic. Namely
> impredicativity.
>
> But hirarchical definitions work well. And
> somebody can also get away with the function
> symbols, and replace them by predicate symbols.
>
> But I am somehow diverting from the original
> topic of interpolation.

And you're also somehow diverting from the obvious
question: If P and Q are sentence symbols in
propositional logic, alpha is P −> P and
beta is Q −> Q, then exactly what gamma uses
only sentence symbols that appear in both
alpha and beta?

(Not that FOL has any relevance to the original
post, but if we do decide we're talking about
FOL, and if we do decide to call unary predicates
sentence symbols, exactly how does that change
anything? Exactly the same question arises.)

> Bye

David C. Ullrich wrote:

> On Wed, 12 Apr 2006 13:29:30 +0200, Jan Burse <janburse@xxxxxxxxxxx> wrote:
>
> > Hi
> >
> > David C. Ullrich wrote:
> >
> > > On 11 Apr 2006 03:36:16 −0700, "Li Yi" <liyi.cn@xxxxxxxxx> wrote:
> > >
> > > > If alpha |= beta, then there is some gamma all of whose sentence
> > > > symbols occur in both alpha and beta and such that alpha |= gamma |=
> > > > beta.
> > >
> > > This is obviously false.
> > > Hint: The weaker statement "If alpha |= beta, then there is some
> > > gamma all of whose sentence symbols occur in both alpha and beta"
> > > is obviously false.
> >
> > Depends on what one understands by sentence symbols.
>
> The subject line specifies _propositional_ logic.
> There's a perfectly standard notion of "sentence
> symbol" in propositional logic
>
> > If for example sentence symbols means variables, function
> > symbols and predicate symbols,

and none of these exist in propositional logic.

These things do of course exist in predicate logic.
Calling them "sentence symbols" seems like maximally
strange terminology; the things that they "represent"
are not sentences.

> then both of them are true.
>
> Let S(.) denote these symbols from ..
>
> The if alfa |= beta, then there should be a gamma with
> S(gamma) subset S(alfa) intersect S(beta). Namely take
> the gamma=false for example. Here S(gamma)={}.
>
> If additonnaly it should hold alfa |= gamma and gamma |=
> beta,
> you end up with craigs interpolation theorem.
>
> http://www.cl.cam.ac.uk/~tjr22/doc/argTalk20051109.pdf

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David C. Ullrich

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David C. Ullrich
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