Re: H. Enderton's proof that theory of natural numbers with successor admits elimination of quantifiers

Source: http://sci.tech−archive.net/Archive/sci.logic/2006−06/msg00395.html

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Fri, 16 Jun 2006 07:14:42 −0500

On 15 Jun 2006 07:37:37 −0700, "recordmymind" <recordmymind@xxxxxxxxx>
wrote:

> Hi David,
>
> Anyway, I took some pictures of the pages in question and the links are here:
>
> pg 191: http://www.flickr.com/photos/48537658@N00/167696412/
>
> pg 192: http://www.flickr.com/photos/48537658@N00/167698616/
>
> I hope the links work.

Aargh. Next time the thing to do is to get an actual web site that you can simply edit, and add the images to it with IMG tags. Or simply upload the files to the server and post a link to the file iteself. I can't find a way to save the image when you put it on this flickr thing (and I need to save the image so I can view it at about three times the size it appears there).

Ok, printscreen/paste/save worked. Back to the math:

The next time someone asks to look again you should look again, much more carefully! The next time someone asks whether there's an "Ex" that you're not telling us about you should look carefully for Ex's, and say _yes_ if one is there!

Enderton does _not_ say that S^m x = t can be replaced in this theory by t not equal to 0 /\ .../\ t not equal to S^(m−1) 0.

To understand a typical proof you need to read the whole thing. At the start he says something about what sort of formula he's

going to replace with a quantidier−free formula. The formula P
he's going to replace has the form Ex alpha. Then in the rest
of the proof he doesn't write down P again, he just discusses
the formula alpha. That formula alpha is _inside_ P − it's
P that he's talking about eliminating the quantifiers from.

In particular, the reason that what he says in one case is
right is that if x does not appear in t then Ex S^m x = t
is equivalent to t not equal to 0 /\ .../\ t not equal to S^(m−1) 0.

> recordmymind wrote:
>
>> Hi David,
>>
>> Quick reply. I read your post and suddenly things made sense. With some
>> quick reasoning, I have verified that what you said about the following
>> is true:
>>
>> Ex S^m x = t is equivalent to t not equal to 0 /\ .../\ t not equal to
>> S^(m−1) 0
>>
>> I think you are absolutely correct regarding the equivalence above.
>> Also, yes, t is a term where x does not appear.
>>
>> Anyway, I'll try to scan the relevant pages and link to them, if not by
>> tomorrow, then by the coming Wed.
>>
>> I'm quite relieved now because this was a stumbling block for the
>> longest time and I could not read on further in the book cos I was
>> stuck at this proof. And this is the second or third time I am trying
>> after a couple of years cos I still hope to achieve my ultimate goal of
>> understanding incompleteness.
>>
>> Thanks a lot, David! Your questions made a big and positive difference
>> to me! I feel ready and rejuvenated to press on with the book! :−)
>>
>> David C. Ullrich wrote:
>>
>>> On 14 Jun 2006 09:54:52 −0700, "recordmymind"
>>> <recordmymind@xxxxxxxxx>
>>> wrote:
>>>
>>>> Dear David,
>>>>
>>>> Thank you for taking the trouble to reply.
>>>> I'm sorry for what is a
>>>> quick and careless reply from me.
>>>>
>>>> Regarding the definition of "replace", there

> is no definition of it in the book. I checked the index to be sure. What follows will make clear whether I have any understanding at all of the subject matter (I'm not sure myself!). I gather that " alpha can replace beta" means that in a model A, any function s from the variables in the language to the domain of model A would satisfy this formula: alpha <−−> beta, i.e. make alpha <−−> beta true in A. In other words, either both alpha and beta are true in A or they are both false.
>
> Regarding:
>
> S^m x = t being replaceable by t not equal to 0 /\ .../\ t not equal to S^(m−1) 0
>
> I swear I checked it again and I stared at what I typed with what was printed on the book. The reason Enderton gives for the longer formula being able to replace the shorter one is "the solution for x must be non−negative".
>
> Nonetheless, I still allow for the possibility that either my eyes have deceived me or I have typed wrongly. If you could give me your email, I will scan the relevant pages and email them to you within 2 days' time.

Before you do that, look again. Is it really S^m x = t, or is it "Ex S^m x = t"? I ask because the second one _is_ equivalent to
"t not equal to 0 /\ .../\ t not equal to S^(m−1) 0".

(Also it's not clear what your version might have to do with quantifier elimination, since there are no quantifiers on either side. Also the comment about "the solution" is a little hard to follow unless there's an "Ex" there.)

Come to think of it, I bet there's also a condition that says something like "if x does not occur in t" that you haven't told us about, right? (Here I'm assuming that t is supposed to be a term...)

If you do end up scanning something don't email it to me.
(i) It probably wouldn't survive my large−file filter anyway
(ii) A better idea is to post the scan on a web site and
then post a link to it here − then anyone who's curious
can see it.

I appreciate your kindness and patience in
wanting to help me see my
way through the proof. Thanks.

David C. Ullrich wrote:

On 13 Jun 2006 07:15:37
−0700, "recordmymind"
<recordmymind@xxxxxxxxx>
wrote:

David C.
Ullrich
wrote:

On
12
Jun
2006
11:11:14
−0700,
"recordmymind"
<recordmymind@xxxxxxxxx>
wrote:

I'm
looking
for
someone
who
is
familiar
with
the
second
edition
of
Enderton's
A

Mathematical Introduction to Logic. I'm having difficulties understanding one of Enderton's proofs and would be extremely grateful if someone here could help me with my difficulties.

In page 191, Theorem 31 G states that the theory of the natural numbers with successor admits elimination of quantifier.

First question, on

page 192, I do not understand why 0=0 can replace the entire formula in Case 1. Is it because in Case 1,

A_s |= (0 = 0 <---> Exists x (alpha_0 /\ ... /\ alpha_q))

where each alpha_i is a formula of this form: ~ (S^m x =

S^n u)?

If p and q are two formulas, exactly what does it mean to say that "p can replace q" here?

recordmymind: Yes, that is a question I also asked when I read that particular page I mentioned.

??? You need to get the definitions of the terms straight first...

Note that I'm just guessing here. When we say that a theory admits elimination of quantifiers surely that means that any formula is equivalent to a quantifier−free formula. So surely

if a theory implies that two
formulas are equivalent then
that means that one formula
"can replace" the other.
(???)
What else could it mean?

Second question, why is it that in Case 2, alpha_0 is replaceable by t not equal to 0 /\ .../\ t not equal to S^(m−1) 0? Note: alpha_0 is S^m x = t

Is that _exactly_ what alpha_0 is? I tend to doubt it.

recordmymind: well, perhaps you could check out the book reference and verify it for yourself. Look forward to your response after you have read those two pages I mentioned. Thanks.

The two formulas you cite here are not equivalent. But with a tiny change to one of them they become equivalent. Perhaps you could look again, and see whether you actually typed them both correctly.

I thank some kind soul in advance.

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David
C.
Ullrich

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David C. Ullrich

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David C. Ullrich

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David C. Ullrich
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