Re: Set Theory: Should You Believe

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- *From*: "Kevin Karn" <kkarn@xxxxxxxx>

- *Date*: 6 Jul 2006 04:03:53 −0700

R. Srinivasan wrote:

<snip>

> I would
> also suggest to NW not to waste time burying himself too deeply in the
> abstruse stuff discussed in the FOM newsgroup; most of the people there
> are already committed to this or that viewpoint and will not accept any
> questioning of the status quo. In other words, these people are
> anything but objective in analysing and questioning the sorry state of
> the foundations of logic and mathematics as exists today. However I
> will not deny that there are some interesting discussions on that list
> from time to time. But if at all anyone successfully questions the
> existing foundations and proposes an alternative, it will have to be an
> amateur with no stakes in the status quo and with no obligation to
> his/her colleagues to justify the status quo; the FOM list does not
> meet this criterion.

This is a question from the sociological angle, but I'm curious to know what you think. Why, in your opinion, is the orthodoxy in set theory etc. so entrenched? Why is the idea of questioning/rejecting infinity so threatening? What exactly is the stake which people have in the status quo? What do they have to lose if the status quo is upset? Why is the resistance so fierce? (Sorry for so many questions. :−) I think you can see what I'm driving at.)

> Mathemaitcs is about producing proofs
> and proofs do require axioms as starting points. To say that
> mathematics does not require axioms is the same is saying that
> mathematics does not require formal systems or even logic itself. But
> that will quickly lead us into contradictions and just plain confusion
> as well.

When NW said that "You don't need axioms", I understood him to be saying something more nuanced −− i.e. that "You don't need set theory,

or the axioms of set theory, to do mathematics."

As he said:
" Whenever discussions about the foundations of mathematics arise, we pay lip service to the `Axioms' of Zermelo−Fraenkel, but do we every use them? Hardly ever. With the notable exception of the `Axiom of Choice', I bet that fewer than 5% of mathematicians have ever employed even one of these `Axioms' explicitly in their published work. The average mathematician probably can't even remember the `Axioms'. I think I am typical−−−in two weeks time I'll have retired them to their usual spot in some distant ballpark of my memory, mostly beyond recall.
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It's very clear that you don't need set theory or the axioms of set theory to do mathematics. After all, virtually the entire body of pre−20th century mathematics was developed without set theory, or axioms of set theory. Even today, mathematicians like NW do productive work without even knowing set theory, or the axioms of set theory, and this clearly shows that the whole field is an unnecessary appendix.

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