Re: Set Theory: Should You Believe

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Kevin Karn wrote:
[...]

> This is a question from the sociological angle, but I'm curious to know what you think. Why, in your opinion, is the orthodoxy in set theory etc. so entrenched? Why is the idea of questioning/rejecting infinity so threatening? What exactly is the stake which people have in the status quo? What do they have to lose if the status quo is upset? Why is the resistance so fierce? (Sorry for so many questions. :−) I think you can see what I'm driving at.)

Well, your questions have confounded me too, and I don't have clear answers. I guess it is partly human nature to be intolerant of dissent, but that doesn't explain everything. I think it is a big mistake made by the academicians to plunge headlong into set theory and infinitary mathematics, ignoring the misgivings of geniuses like Poincare, Weyl, Brouwer and many others in the early part of the twentieth century. I guess the dominance of great mathematicians like Hilbert ensured that these objections were ignored. The institutionalization of research produced many professionals who had no option but to churn out one paper after another in the mainstream research areas, with virtually zero support for any significant pursuit of alternative foundations.
All I can say is that this is just plain wrong. Whoever it was that decided the funding priorities over the last several decades (ever since the early 1900's) should have had the vision to support and encourage alternative viewpoints that may have flatly contradicted the status quo. Why can't these dissenters co−exist with the mainstream guys and get funding? That would have led to a healthy balance that is clearly absent today. It is amazing that today research into foundations is a very low priority for mathematicians and even logicians. I would recommend that you look at a recent paper by Lee Smolin on these issues in the context of theoretical physics (I forget the title of the paper, but you can search his website and get it).

[..]

> When NW said that "You don't need axioms", I understood him to be saying something more nuanced −− i.e. that "You don't need set theory, or the axioms of set theory, to do mathematics."

In the passage that I quoted in my previous post, what NW said was that mathematics did not need axioms at all, period (whether set theoretical or otherwise). That viewpoint presumes a certain 'reality' that mathematcians have to take for granted and simply make definitions to access that reality, in their proofs. This is not right −− there is no way for us humans to access any such reality (certainly about infinitely many objects, such as, natural numbers). This alleged 'reality' is just a figment of our imaginations −− which is what I would call axiomatic declarations in the human mind. i.e., these "truths" have their (temporary) existence in the human mind as axioms. Using the rules of inference in some system of logic that the human mind accepts, one can deduce theorems from these axioms. Ultimately all the theorems of such a theory are therefore just declarations made by the human mind. This is the position that I take in my work on the logic NAFL, and you can show why infinite sets are not acceptable in NAFL theories (but infinite proper classes, which are not mathematical objects (i.e., sets) are acceptable in NAFL theories).

> As he said:
> " Whenever discussions about the foundations of mathematics arise, we pay lip service to the `Axioms' of Zermelo−Fraenkel, but do we every use them? Hardly ever. With the notable exception of the `Axiom of Choice', I bet that fewer than 5% of mathematicians have ever employed even one of these `Axioms' explicitly in their published work. The average mathematician probably can't even remember the `Axioms'. I think I am typical−−−in two weeks time I'll have retired them to their usual spot in some distant ballpark of my memory, mostly beyond recall.
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> It's very clear that you don't need set theory or the axioms of set theory to do mathematics. After all, virtually the entire body of pre−20th century mathematics was developed without set theory, or axioms of set theory. Even today, mathematicians like Wildberger do productive work without even knowing set theory, or the axioms of set theory, and this clearly shows that the whole field is an unnecessary appendix.

How would you define real numbers, for example, without set theory? Wildberger seems to assert in his paper that as long as we have a finitely stated rule for generating the n'th term in a Cauchy sequence of rationals, there is no need to consider a real number as an infinite object. I don't agree with this. For starters, you need to talk about an "arbitrary" natural number n to generate this rule −− what is this n? It only makes sense to consider n as a variable that can take on *any* of infinitely many possible values; in my view this presumes the existence of an infinite class of natural numbers. Similarly the range of the sequence is another infinite class of rationals described by the rule for generating r_n, the n'th rational in the list. Wildberger rejects this and says that a function does not need to be considered as having an infinite domain and an infinite range, as long as there is a finite rule to generate the function. My own view is that the existence of these infinite classes themselves is not the problem; it is quantifying over these classes, i.e., formally referring to infinitely many such infinite classes in a formula, that constitutes infinitary reasoning, tacit in set theory or almost any modern mathematical theory. This is what my logic NAFL avoids and one can still do real analysis in NAFL as I pointed out in my previous post. Wildberger's objection to an *arbitrary* real number x (see my previous post) can now be rationalized as an objection to quantifying over all the possible real values that x can take, since each real is an infinite object. Otherwise what precisely is Wildberger's objection to an arbitrary real number, since he already accepts arbitrary natural numbers?

Regards, RS

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