Re: PC(1): An introductory formal logic

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Sat, 08 Jul 2006 05:43:53 −0500

On 7 Jul 2006 09:20:22 −0700, "George Dance" <georgedance04@xxxxxxxx>
wrote:

> David C. Ullrich wrote:
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>> On 6 Jul 2006 06:33:14 −0700, "George Dance"
>> <georgedance04@xxxxxxxx>
>> wrote:
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>>> David C. Ullrich wrote:
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>>>> On 5 Jul 2006 19:10:25 −0700, "George
>>>> Dance" <georgedance04@xxxxxxxx>
>>>> wrote:
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>>>>> Frederick Williams wrote:
>>>>>
>>>>>> George
>>>>>> Dance
>>>>>> wrote:
>>>>>>
>>>>>>> An
>>>>>>> axiom
>>>>>>> is
>>>>>>> a
>>>>>>> formula
>>>>>>> that
>>>>>>> is
>>>>>>> true
>>>>>>> in
>>>>>>> every
>>>>>>> interpretation.
>>>>>>
>>>>>> Your

muddling the syntactic ("An axiom is a formula") with the semantic ("true in every interpretation").

There is absolutely no distinction, in this system, between the semantic and the syntactic. Why do you think that there has to be?

Um. You said that the idea is to make it easier to learn real formal logic later.

Yes, indeed. You do that by not trying to dump everything on them at once, as one more or less has to do a one−semester introductory college course.

In PC(1), there is no distinction between semantic and syntactic: the truth table is the proof method, and the semantic interpretations are the lines (m, m et al) of the truth table. It is only when the student learns PC(1) and PC(2), and goes on from there to a formal deductive system − where what is provable in the system may be different from what's valid by the truth table, that the semantic/syntactic distinction even comes up. So that is precisely the point to introduce it; when it makes some sense to the students, not at the beginning where it means nothing.

This one is _also_ exactly how arithmetic
is taught, in some
other schools. Earlier I mentioned the
common practice of
introducing 1 + 1 = 2 early on, but leaving 2
+ 2 = 4 undefined
because we don't want to confuse them with
"4". There are of
course other schools, where the idea of
leaving 2 + 2 undefined
is regarded as a bad idea. So instead in first
grade they define
x + y = 2 for _every_ x and y. There's no
distinction in that
system between 2 and 4... some people think
there should be,
I don't see why.

Oh, come on. What you're describing sounds like exactly
what I'm
proposing with PC(1); a limited arithmetic (restricted to the
number
set {0,1,2}). Because of that limit, it's as useless for
reasoning
mathematically as PC(1) is for reasoning logically. But that
isn't its
point; the point is to familiarize the students with the signs
and
symbols and the way to use them.

Then why do you insist that the symbol should be |− when
what you're really talking about is |= ?

|− isn't an operator of the system; it's a symbol used to talk about
the system.

Huh? Yes, we all know that − what does that have to do with the
question of whether it should be |− or |=.

Which symbol one uses, in this case, makes absolutely no
difference; there is no difference between a proof using a truth table
test, and PC−validity. I already told you that.

Uh, yes, you told me that. In actual logic there is a huge
difference − if there were no difference then the Soundness

and Completeness Theorem, to the effect that |− and |= are
actually equivalent, would have no significance.

In fact in your system there is literally no difference, but
only because you've _defined_ |− to mean what everyone else
would call |=. If for whatever reason you're _going_ to insist
on teaching kids about these things, the idea that you
should use the wrong symbol, given that there's no compelling
reason not to use the right symbol, seems like a bad idea.

I mean if you were writing a book on arithmetic there's
also no reason you could not define "−" to mean what most
people call "+", and then teach them that 2 − 2 = 4. But
it doesn't seem like a good idea.

> But, since you think it's quite important, I'll do something for you:
> If I write a book or article on PC(1), I'll use |=, and be sure to
> thank you for your help in the acknowledgements.

Fabulous. Make certain to let us know when it's published.

> > Which is the point of PC(1) as well.

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>
> David C. Ullrich

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David C. Ullrich
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