

## Re: A question about FOL theories and models

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*Source:* <http://sci.tech-archive.net/Archive/sci.logic/2006-08/msg00792.html>

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- *From:* [tchow@xxxxxxxxxxxxxx](mailto:tchow@xxxxxxxxxxxxxx)
  - *Date:* 19 Aug 2006 15:08:39 GMT
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In article <[rkmFg.421858\\$Mn5.15315@pd7tw3no](mailto:rkmFg.421858$Mn5.15315@pd7tw3no)>, Nam Nguyen <[namducnguyen@xxxxxxx](mailto:namducnguyen@xxxxxxx)> wrote:

ZF(C), after all, is just one theory out of infinite number of 1st order ones. (Formally, FOL framework never insists that we have to know about ZFC to formalize a different theory.)

Assuming we've formalized a theory G of "geometry", how could we prove that the 5th "postulate" – as an axiom – is unprovable in G, without mentioning anything about ZF(C)? In other words, how could we possibly come up with a specific model of G in which the 5th is false? Thanks.

The answer to your question as you've stated it is, we construct a non-Euclidean geometry, such as an elliptic or a hyperbolic geometry. There is no need to say anything about ZF(C) in order to construct such a geometry.

I suspect that your real question, however, is not the one that you've stated. Implicitly, perhaps, you feel that you can't "do mathematics" unless you state ahead of time some axiomatic system in which the mathematics you want to do is to be formalized. I believe that this is a serious misconception about the nature of mathematics and of formalization, but I also know from experience that trying to convey this point on USENET tends to lead to interminable and fruitless discussions, so I'll sidestep it for the moment. Your question might therefore be, can we formalize the construction of a non-Euclidean geometry without using set theory? Or, if set theory seems to be unavoidable, is ZF(C) required, or are there weaker set theories that suffice?

For these kinds of questions, I do not believe that there is anything special about the construction of a non-Euclidean geometry, as opposed to any other piece of mathematics, e.g., the construction of finite simple groups or expander graphs. The general answer in all these cases is probably going to be something like, "Set theory is probably the most natural language in which to formalize the proof in question, although if you insist on dispensing with set theory, you can probably choose some other general foundation for mathematics, and that will

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work just as well. If you use set theory, then the full power of ZF(C) is almost certainly not needed." There may be exceptions to this general rule, but the construction of non-Euclidean geometries is so "low-powered" that it can almost certainly be formalized in any nontrivial theory you care to name.

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The range of our projectiles— even ... the artillery— however great, will never exceed four of those miles of which as many thousand separate us from the center of the earth. — Galileo, Dialogues Concerning Two New Sciences

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