

Knowledge in Action (Reiter) – example 2.1.1

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Hi all,

I have just started reading Raymond Reiter's "Knowledge in Action: Logical Foundations for Specifying and Implementing Dynamical Systems", and am finding it hard to understand an early example. The author is explaining why first-order logic is insufficient to define transitive closure of a graph, G , and says that the following "naive" definition is incorrect:

$$T(x, y) \iff G(x, y) \vee (\exists z).(G(x, z) \wedge T(z, y)).$$

(Hopefully that is readable: E is existential quantification here).

The author then describes a counter-example (example 2.1.1):

"Consider the directed graph with two vertices a and b , and with a single directed edge $G(b, b)$. Consider a structure with universe $\{a, b\}$ that interprets G as $\{(b, b)\}$ and T as $\{(a, b), (b, b)\}$. In other words, the structure represents the graph G (by declaring that there is an edge from b to b , and no other edges), and it assigns true to $T(a, b)$ and $T(b, b)$. It is easy to check that this structure is a model of the above naive definition for transitive closure. ..."

I must be missing something in the semantics, because I don't see how a model of T given the above definition can allow $T(a, b)$ to be true, as there is no edge in G that involves a at all. Any help greatly appreciated.

I'd like to make sure I understand this point fully before I move on, as it seems crucial to Reiter's argument for second-order logic.

-- Neil

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