Re: Knowledge in Action (Reiter) − example 2.1.1

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- *Date*: Sun, 11 Mar 2007 18:57:16 +0000 (UTC)

On Sun, 11 Mar 2007 17:40:38 GMT, Neil Madden <nem@xxxxxxxxxxxxx> said:

> Hi all,
>
> I have just started reading Raymond Reiter's "Knowledge in Action: Logical Foundations for Specifying and Implementing Dynamical Systems", and am finding it hard to understand an early example. The author is explaining why first−order logic is insufficient to define transitive closure of a graph, G, and says that the following "naive" definition is incorrect:
>
> T(x, y) <=> G(x, y) \/ (Ez).(G(x, z) /\ T(z, y)).
>
> (Hopefully that is readable: E is existential quantification here).
>
> The author then describes a counter−example (example 2.1.1):
>
> "Consider the directed graph with two vertices a and b, and with a single directed edge G(b, b). Consider a structure with universe {a, b} that interprets G as {(b, b)} and T as {(a, b), (b, b)}. In other words, the structure represents the graph G (by declaring that there is an edge from b to b, and no other edges), and it assigns true to T(a, b) and T(b, b). It is easy to check that this structure is a model of the above naive definition for transitive closure. ..."
>
> I must be missing something in the semantics, because I don't see how a model of T given the above definition can allow T(a, b) to be true, as there is no edge in G that involves a at all.

T(a,b) is simply true by stipulation in the new structure. The problem is that the new structure satisfies the naive definition. The naive definition is supposed to pick out, for any graph <V,E>, its (unique) transitive closure. What Reiter's example shows is that the naive definition lets in too much. In particular, the TC of the directed graph <{a,b},{<b,b>}> is just itself. However, the graph <{a,b},{<a,b>,<b,b>}> also satisfies the naive definition. (I do find his argument for this above a little unclear −− he uses "G" both as a name for a graph and as a binary predicate.) Hence, the naive definition has "unintended" models, which Reiter goes on to argue can only be ruled out in second−order logic.

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