Re: infinitely many nn's = infinite nn's?

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On Mar 13, 2:32 pm, Phil <toob−head...@xxxxxxxxxxxxx> wrote:

> However, do you think, YOURSELF, that infinitely many FINITE values REALLY CAN sum to a finite value? Remember, the fact that {1/2, 1/4, 1/8, 1/16, ...,} sums to 1 after INFINITELY many elements could simply be a result of the fact that only finitely many (potential rather than actual infinity) of these elements are finite, and that infinitely many of them are infinitesimals.

The answers to your questions are given in the logic NAFL; see

http://arxiv.org/abs/math.LO/0506475

I have already explained in sci.logic threads as to how NAFL deals with and resolves Zeno's paradoxes. In particular, to answer your question briefly, there are no infinite sets in NAFL, and quantification over infinite classes is banned. For example, you question above as to "How many" intervals are present in the following sequence of closed real intervals:

{[0,1/2], [1/2,3/4], [3/4,7/8], ...}

is an *illegal* question in NAFL, i.e., it cannot even be formulated because the above sequence of intervals is not even a legal entity in NAFL (each of these is an infinite object and to make the sequence of such intervals a legal entity, you need to quantify over infinite objects (intervals of reals) which is precisely what you cannot do in NAFL). There are no infinitesimals in the NAFL version of real analysis.

Zeno's paradoxes are resolved because there are no open intervals in the NAFL version of real analysis. The above sequence, with no smallest interval, can only be possible if open intervals of reals exist. In the NAFL version of real analysis, the above sequence is represented indirectly without any quantification over infinite entities like reals or intervals of reals; one then finds that this resulting NAFL sequence *must* include an interval of zero length, [0,0]. So you cannot assert that there is a sequence of such "infinitely many finite, non−zero intervals" because the sequence must include an interval of zero length. Secondly you cannot ask "how many" closed intervals are present in the NAFL version of such a sequence (which includes the interval [0,0]) because that amounts to quantification over intervals of reals, whereas NAFL does not even permit quantification over reals (each of which is an infinite object), let alone intervals of reals.

In order to clearly understand and appreciate the NAFL resolution of Zeno's paradoxes, you need to understand NAFL from first principles. Then you will accept that there is no largest natural number or infinite natural number. But the NAFL real line must include an object corresponding to +− 00. So if you consider the sequence of real numbers

{0.0, 1.0, 2.0, ...}

it *must* include +00. Here it is important that each of the reals in the above sequence is an infinite object and the entire sequence is *constructed* (indirectly) in NAFL without any quantification over reals.

Similarly the sequence of rationals

{1, 1/2, 1/4, 1/8, ...}

does not have a last element (you can directly quantify over rationals because each of these is a finite object). But if you consider the corresponding sequence of *reals*

{1.0, 0.5, 0.25, 0.125 ....}

then this sequence *must* include the real number 0.0.

According to you this sequence *must* include infinitesimals, but that does not hold in NAFL (or you can take zero to be the only legal infinitesimal in NAFL; there are no non−standard ("infinite") natural numbers in NAFL. In fact nonstandard analysis does not really resolve Zeno's paradoxes in the way that you imagine; it will generate its own set of equivalent paradoxes.

In short, Zeno's paradoxes are resolved because the very statements of the paradoxes are only possible if open (or semi−open) intervals of reals exist, but these are illegal in NAFL. Secondly quantification over infinite entities is banned, so you cannot ask the questions that lead to paradoxes even concerning entities that are legal in NAFL (like closed intervals of reals).

> And all it would take for that to be true

> would be to use ACTUAL rather than potential infinity when determining
> not only the number of natural numbers, but also the maximum value that
> natural numbers can have. As I said elsewhere, if you use either
> potential infinity OR actual infinity for BOTH the number of numbers and
> the maximum value of a number, this particular paradox instantly
> vanishes. Does that even slightly suggest to you that potential and
> actual infinity might NOT be the same thing?
>
> Do you think, in situations like these, that SOMEONE −− preferably a
> good mathematician who really knows what he's doing, but someone, in any
> case −− should at least look into the possibility that the proofs rest
> on incompatible premises? Or do you believe that, despite all the errors
> made concerning infinity for 2500 years, that starting about 150 years
> ago, we got everything right, everything perfect and error−free, and
> there's no need to even verify that no problem exists, no need to even
> investigate any paradoxes? It is as much the implicit claim that
> mathematicians obtained godlike perfection toward the end of the 19th
> century, as much as anything, that does amazes me −− or disgusts me −−
> about this group.

The resolution of your difiiculties is already available. You haven't heard about it because of the extreme reluctance, if not outright unwillingness, of academicians −− in particular, logicians and phillosophers −− to even acknowledge the existence of my work (let alone analyze it, criticize it , improve it or dismiss it, etc.).

Regards, RS

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