Re: Minimal set theory for model theory?

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* *From*: LauLuna <laureanoluna@xxxxxxxx>
* *Date*: 26 Apr 2007 08:21:24 −0700

On Apr 26, 12:43 am, george <gree...@xxxxxxxxxx> wrote:

> On Apr 24, 11:49 am, LauLuna <laureanol...@xxxxxxxx> wrote:
>
> > A previous post hasn't appeared.
>
> I too had a previous post fail to appear.
> I wasn't bothered by it at first because I am not
> actually competent to answer the question being posed.
> But the discussion will have to begin on this lower level
> until Someone Who Knows deigns to Speak.
>
> > Well, take into account Orayen's paradox.
>
> That's easier said than done, in English anyway.
> If you google "Orayen's Paradox" you will not get any
> hits outside this thread. If you Google Orayen and Paradox
> disjunctively then you will find that Raul Orayen wrote a lot
> of things about paradoxes, but most of them are in Spanish.
>
> > It is caused by the fact that we use FOL to formalize set theory
>
> Right.
>
> > and set theory to interpret FOL formulae.
>
> But not necessarily; the content of my missing post was that
> the completeness theorem makes interpretation irrelevant.
> FOL formulae simply don't need to be interpreted at all.

Although the typical definition of logical consequence under this paradigm is via satisfaction "in all models", the Completeness theorem is a demonstration that we have the (extensionally) identical notion of consequence via synactic inference rules. In other words, FOL *has* a model theory, but it doesn't *need* one.
If the model theory gets paradoxical or otherwise embarrassing then the obvious solution is simply to throw out the bathwater of model theory and deal thereafter with the baby of proof theory.

> The paradox shows that there is no standard model for first order set theory and derives this from Cantor's paradox: set theory is supposed to speak about any set,

And, more to the point, logical consequence is defined in terms of any and every model, so it, too, would need to speak about every set (the point being that every set COULD be [the domain of] a model of every theory that didn't force a bigger cardinality).

> so that the universe of discourse of the corresponding interpretation should be the set of all sets, which does not exist.

Somebody needs to read Zermelo's original 1908 paper.
It says basically that we have a domain of objects B and that among these are the sets. It does NOT say that B has to or needs to be a set; indeed, with the axiom of foundation, it absolutely cannot be (it would have to contain itself if it were a set). The rebuttal of this objection is that THE CLASS of all sets DOES exist, and the general dictum should be that the domain of a model has to be a CLASS. Whether the class is proper or is a set is not even important. From the viewpoint of the theory INTERNALLY,
the universal class will ALWAYS be proper, but this does not prohibit it from becoming a set in a meta−theory.

> This we usually prove from Powerset and Cantor's theorem.

> So, contrary to what it might seem, model theory for FOL could require quite a bit of set theory.

Well, at a bare minimum it requires a class theory as opposed to the ZFC conceit that proper classes don't exist. But ZFC's failure to understand its own whole domain IS NOT a paradox.

Ordinarily we understand that the domains of our models are sets. But if you introduce proper classes into the picture, well, then we surely want to interpret set theory as being about all classes. So, again we would have no model for it, unless we admit the existence of a universal class.

Since Raúl Orayen was Argentinian, you may be right that most references to his paradox are in Spanish. It seemed that Orayen had some relationship to Quine; nevertheless I cannot provide any reference in English, at least no more than what you could find on the web.

A related difficulty lies in the interpretation of some FOL theorems like Thomson's theorems: −Ex Ay Rxy <−> −Ryy. Note that this is closely related to Russell's paradox.

Take R as the 'aboutness' relation among propositions, so defined: p is about q iff there is a proposition r equivalent to p and the subject of r either denotes q or a class to which q belongs. The we interpret the theorem as:

(1) there is no proposition exactly about all propositions not about themselves and only them.

And consider:

(2) all propositions not about themselves share the property that there is no proposition exactly about them.

Well, (2) is equivalent to (1) and seems to be exactly about all propositions not about themselves.

Consequently there is an interpretation of the theorem, namely (1), that seems to violate the theorem itself!

I think we can only escape this by imposing a restriction on the range of the quantifiers in the interpreted theorem: the universe of discourse cannot include the interpreted theorem itself.

So, again, paradoxes can force us to consider sophisticated issues of set theory in order to account for the interpretation of FOL formulas.

Regards

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