

Re: Cantor's circular "proof" that evens = integers

Source: <http://sci.tech-archive.net/Archive/sci.logic/2007-05/msg00561.html>

- *From:* Aatu Koskensisilta <aatu.koskensisilta@xxxxxxxxxx>
 - *Date:* Thu, 24 May 2007 13:50:53 GMT
-

On 2007-05-19, in sci.logic, herbzet wrote:

I'll take your word for this, since I am struggling with the concept of a consistent theory proving a "false" proposition. A consistent theory has models, and what it proves is true in all of them. If a consistent theory proves a false proposition, it must be that the structure in which we are interpreting the theory is not a model of the theory.

Indeed. If a theory proves a false arithmetical statement, for example, then the naturals (possibly with some relevant additional structure, sometimes embedded in a larger structure) are not a model of the theory. If the theory is intended as a formalization of some body of knowledge about naturals, so that we might speak e.g. of a number theoretical theorem being provable or unprovable in it, proving false arithmetical statements is just as bad as being inconsistent. Another point I wished to make was that people usually obsess over consistency simply because they think a theory being consistent or inconsistent is somehow more concrete or objective a state of affairs than it being unsound for some class of sentences. We can counter this by noting that a theory T proving "T is inconsistent" is equally "objective". Of course, people who have sentiments like this are usually not consistent in applying their doubts, and do not similarly doubt the objectivity of statements of form "the algorithm A terminates on all inputs" and such like.

It is also important to note that that a consistent theory might prove false statements is a mathematical observation no more problematic than, say, the existence of non-Archimedean fields or non-Abelian groups.

--

Aatu Koskensisilta (aatu.koskensisilta@xxxxxxxxxx)

"Wovon man nicht sprechen kann, darüber muss man schweigen"
– Ludwig Wittgenstein, Tractatus Logico-Philosophicus

.