Re: Cantor's circular "proof" that evens = integers

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Aatu Koskensilta says...

> Another point I wished to make was that people usually
> obsess over consistency simply because they think a theory being consistent
> or inconsistent is somehow more concrete or objective a state of affairs
> than it being unsound for some class of sentences. We can counter this by
> noting that a theory T proving "T is incosistent" is equally "objective".

This might be obvious to most people, and so not worth saying,
but I want to point out that when someone says that for some
formula Con,

Con formalizes the metatheoretic claim that T is consistent

this claim must be relative to a particular *interpretation*
of the language in which Con is expressed. To formalize consistency,
we concoct a formula along the lines of

forall y, forall z, forall w
Negation(y,z) & And(y,z,w)
−> ~ forall x, Proof(x,w)

where Negation(y,z) is a formula meaning
"z is the code of the negation of the
formula whose code is y", and And(y,z,w) is a formula meaning
"w is the code of the conjunction of the two formulas whose codes
are y and z" and Proof(x,w) is a formula meaning
"x is a code for a proof of a formula whose code is w".

But what forces Negation(y,z) and And(y,z,w) to be about formulas?
What forces Proof(x,w) to be about proofs? You can't determine
syntactically that a formula is about proofs or is about formulas.
You have to establish, in some metatheoretic way, that for the
intended domain, Proof(x,w) is true if and only if x is a code
for a proof, blah, blah, blah. So Proof(x,w) only means proof
relative to that intended domain.

If you take PA and add the axiom ~Con(PA), then the resulting theory has a model. But that model does *not* interpret the formula ~Con(PA) to mean "PA is inconsistent". In that nonstandard model, ~Con(PA) means that there exists an object x with certain properties, but that object is *not* a code for a proof.

Saying that there is a model for PA + PA is inconsistent is misleading. What's really true is that there is a model for PA + that formula which is interpreted as "PA is inconsistent" in the standard model.

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