

Re: Question about extensions of PA

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Herman Jurjus <h.jurjus@xxxxxxxxxx> writes:

Got time for a question? On the FOM list, you posted (a while ago, June 3 '07), the following:

> Richard Heck wrote:

[snip]

>> ... We want a system PA* that adds to PA precisely the
>> instances of: Bew_{PA*}(A) \rightarrow A. I'm sure this can be done,
but, >> as I said, it doesn't seem trivial.

>

> Nothing but a trivial application of the recursion theorem is needed.
> The axioms of the desired theory T will be recognized by a recursive
> function f(x) with index e, where f(x) is defined by

>

> f(x) = 1 if x is the code of an axiom of PA or a code of "Prov_e(A)

> \rightarrow A"

> f(x) = 0 otherwise

>

> where Prov_e(A) is the formalisation of "A is derivable from the set
> of sentences recognized by the recursive function with index e".

In a later post, you also defended this; so you seemed to mean it, too.
But i've become a bit puzzled about it. Please permit me to ask the
following questions:

- 1) What guarantees that the resulting fixed point is consistent?

It obviously isn't. The point is simply to obtain a set of sentences A
with the following properties

A is recursive

A contains the axioms of PA

A contains every instance of "if Prov_A(P) then P", for some
suitable definition of Prov_A

This set is an axiomatisation of the highly interesting set of all

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arithmetical sentences, i.e. the inconsistent theory, as follows from the second incompleteness theorem. Still, producing a set of sentences having these properties is not in itself an utterly trivial task if one does not happen to think of the recursion theorem.

I'm not sure what the rest of your questions actually mean, but perhaps they're based on the misconception that the constructed theory is consistent? The function f is well-defined by the recursion theorem — or the Kleene fixed point theorem as it's also known —, if that's what you're wondering.

2) Are you sure that the function f is a well-defined function from recursive functions to recursive functions?

f in the above is not a function from recursive functions to recursive functions, but rather a function from naturals to naturals (from codes of sentences to $\{0,1\}$), obtained by the recursion theorem. As it results from an application of the recursion theorem, its ancestor might be said to be a function of recursive functions to recursive functions, a recursive functional if you wish, taking as an argument an index e of recursive functions, f being the fixed point of this ancestor function.

I mean, if two descriptions of such functions are equivalent, don't we also require that PA *can prove* they're equivalent, before we can conclude that $f(e_1)$ is equivalent to $f(e_2)$?

I'm afraid I don't quite see what you're asking here.

3) Isn't it a core point of exhaustion theory that PA-extensions like the one asked for can't possibly be r.e.? (Unless they're inconsistent.)

Yep. The axiomatisation is of the inconsistent theory, though a particularly interesting axiomatisation from a certain theoretical point of view. It illustrates the fact that the incompleteness theorem shows not only that formal theories are too weak to prove their consistency, but also that they simply can't, on pain of inconsistency, prove their own consistency even by stipulation, i.e. by adopting it as an axiom, just as a man can't truthfully assert he never speaks of himself.

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"Wovon man nicht sprechen kann, darüber muss man schweigen"

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– Ludwig Wittgenstein, Tractatus Logico–Philosophicus

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