

Re: Question: Given $|X|>0$ and $|Y|>0$, can $X \times Y$ be empty?

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- *From:* Scott <ToaTerra@xxxxxxxxxx>
 - *Date:* Fri, 03 Aug 2007 23:43:57 -0000
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On Aug 3, 4:00 pm, magi...@xxxxxxxxxxxxxxxxxxxxx (Arturo Magidin) wrote:

In article <1186161736.517279.72...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,

Let f be a function $f:S \rightarrow P(S)$.

....

So, you are saying in Prop 4: if p is an element of $P(S)$, then there is a function (with domain where?) whose image includes p .

Better:

If p in $P(S)$, then there exists $f(p) : S \rightarrow P(S)$ such that p is in the range of $f(p)$.

I was trying to define the domain and codomain of any f by the single sentence. I can see that was a mistake. The better sentence should be:

If p in $P(S)$, then there exists $f(s) : S \rightarrow P(S)$ such that p is in the range of $f(s)$.

Proof: If p in $P(S)$, then $(\exists s \in S)((s,p) \in M)$ [Prop 3]. Let $f(s) = p \iff (\exists s \in S)((s,p) \in M)$.

This is lousy. Are you trying to define f as "constant p "? Then DO SO. What you write here is pretty near unintelligible, close to

As a constant function – yes. I thought that is what I was defining: for any p in $P(S)$, there exists a constant function $f(s) = p$. I was showing this function exists because its a subset of M .

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nonsense. Since (s,q) in M for all s in S and all q in $S(p)$, your condition on the right ALWAYS holds, so it does not define a function. Your "function" is in fact "assign to each element of S every element of $P(S)$ " and that is not a function.

No, the other way around: for each p define a function f_p that maps every element of S to one element of $P(S)$. To use your notation below:

(Ap)(let $f_p(s) = \{ (s,p) \text{ in } M \}$)

Perhaps you can educate me on the correct way to say this?

f is functional

No, it is not. Your definition makes $f = M$, and that is in general not a function.

Example: Given $f: \{0, 1, 2\} \rightarrow \{\text{cat,dog}\}$ where $f = \{ (0,\text{cat}), (1,\text{cat}), (2,\text{cat}) \}$ why is this not functional and total?

Let $A = \{ x \text{ in } S : x \text{ not in } f(x) \}$.

What f ? The one you define in Prop 4 for a given p , or the one we

At this point, any f . Since I am trying to make this general, I am not restricting which f .

So, let f be an arbitrary function, $f:S \rightarrow P(S)$. It need not be equal to $f(p)$ for any p in $P(S)$, i.e., it may not be the conditional one we have from Proposition 4.

Correct, it can be any f , not necessarily the one from Prop 4.

Given f , we define A_f to be

$A_f = \{ x \text{ in } S : x \text{ not in } f(x) \}$.

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Okay, I can see this is better.

This is simply Cantor's Argument.

From what I read, Cantor's Argument deals with surjective functions. I

am claiming there are *no* functions with A_f in the range including simple ones constructed with finite sets.

From Prop 4 and the contrapositive, $\sim(Ef)(A_f \text{ in range}(f)) \rightarrow A_f \text{ not in } P(S)$.

Wrong. Proposition 4 was incorrect and does not refer to the same f as proposition 5. Here is your mistake, born out of that LOUSY notation where you use f to mean many different things at the same time.

Can you explain this a bit more. Prop 5 applies to any function, including the function in Prop 4. Given $\sim(Ef)(A_f \text{ in range}(f))$ isn't it true that $\sim(A_{\{f(s)\}} \text{ in range}(f(s)))$?

Proposition 4 says:

$(\forall p)(p \in S(p) \rightarrow (E f(p))(f(p):S \rightarrow P(S), f(p) \text{ is a function, and } (\exists x)(x \in S \text{ and } f(p)(x)=p)))$

Err,no. It says:

$(\forall p)(p \in S(p) \rightarrow (E f(s))(f(s):S \rightarrow P(S), f(s) \text{ is a function, and } (\exists x)(x \in S \text{ and } f(s)=p))$.

Question: Would it have been a whole lot clearer if I constructed the proofs using just the one function from Prop 4?

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