Re: Question: Given |X|>0 and |Y|>0, can X x Y be empty?

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- *Date*: Tue, 07 Aug 2007 18:47:20 −0000

On Aug 3, 5:19 pm, magi...@xxxxxxxxxxxxxxxxx (Arturo Magidin) wrote:

> What would have been better is if you had kept the difference between functions you get from subsets per Prop 4 separate from subsets you get from functions per Prop 5.

Okay, so keeping them separate and using subscripts:

Prop 4A: (AS != emptyset)(Ap)(p in P(S) −> (Eg_p)(g_p:S −> P(S) & p in range(g_p))).
Proof: Let S be any non−empty set. If p in P(S), then let g_p be the constant function g_p: x |−> p where x in S. g_p exists because g_p subset S x P(S) and, by inspection, g_p is functional and total. Furthermore, p in range(g_p). QED.

Prop 5A: (AS != emptyset)(Ep)(p in P(S) −> ~(Ef_p)(f_p:S −> P(S) & p in range(f_p))).
Proof: Let S be any non−empty set. Let A_{f_A} = { x in S : x notin f_A(x) } so A_{f_A} in P(S). Suppose (f_A: S −> P(S) & A_{f_A} in range(f_A)). Then (Ey)(f_A(y) = A_{f_A}). Since A_{f_A} subseteq S, then (Ax in S)(x in A_{f_A} <−> x notin f_A(x)). In particular, when x = y, (y in A_{f_A} <−> y notin A_{f_A}). Contradiction. QED.

Now (hopefully) Prop 4A is a "machine" that, given a subset p, gives me a function g_p with property P and Prop 5A is a "machine" that, given a specific subset A, gives me the result that there are no functions f_A with property P.

Comments/feedback welcome.

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