Re: Continuum hypothesis

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- *From*: george <greeneg@xxxxxxxxxx>
- *Date*: Fri, 17 Aug 2007 08:00:50 −0700

On Aug 16, 11:30 pm, William Elliot <ma...@xxxxxxxxxxxxxxxxxx> wrote:

> On Thu, 16 Aug 2007 djr...@xxxxxxxxxx wrote:
>
>> Yeah it means not CH, screwy keyboard. What is an arithmetic
>> statement? Well that was what I was wondering. I guessed he (Woodin)
>> meant a statement of number theory of some order, first order or
>> second order or whatever.

You were right.

>> Then if it makes sense to say CH is not a
>> statement of any order number theory, then what he said would make
>> sense.

Well, not exactly.
The link you are missing here is that while number theory is normally
written in one axiom−system (PA), and set theory is normally written
in another (ZFC), the latter, being stronger, is capable of being used
to
express the former. So you can TRANSLATE all the statements of
PA *into* set theory −− you can REDEFINE 0,1,<,+, and x, all of which
are from the language of PA, *into* the language of ZFC.
Once you do that, it becomes reasonable to ask how adding new
assumptions
to ZFC will impact the truth, falsity, provability, and disprovability
of
the *ZFC TRANSLATIONS OF* statements that were originally in the
language
of PA.

> CH is a statement about transfinite cardinal numbers.
>
> Arithmetic is about integers and fractions and adding, substracting,
> multiplying and dividing them.

But you are saying this as though "never the twain shall meet". That
is NOT
the case. Set theory is a FOUNDATION. You can do ANYthing in set
theory.
Including and ESPECIALLY arithmetic.

> Number theory is about integers, prime and composite integers and how to find integer solutions for equations.

Yeah, but it might as well just be arithmetic. Essentially, number
theory
invites you to start with arithmetic and add a predicate pronounced
"divides",
meaning "is a factor of", usually spelled | , i.e., (p|q) means
Ez[p*z=q],
and then a function or two counting or summing the numbers that divide
another one.
These "improvements" to the language are actually NOT anything new,
since they
are defined/introduced IN TERMS OF the original basic arithmetic
operations
(they are not NEW more powerful operations).

> Though I've heard about analytic number theory that using analysis to solve number theoric problems, I've never heard the term 'first order number theory'. What does it mean?

Don't panic: simpler than you think. It is NOT a higher more
complicated kind
of number theory. It just means number theory USING FIRST−ORDER
LOGIC,
using syntax and semantics that are about numbers and sets (or
predicates) of
numbers, BUT NOT about Sets OF SETS of numbers (THAT would be *2nd*−
order arithmetic).

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