

Re: Continuum hypothesis

Source: <http://sci.tech-archive.net/Archive/sci.logic/2007-08/msg00397.html>

- *From:* stevendaryl3016@xxxxxxxxxx (Daryl McCullough)
 - *Date:* 20 Aug 2007 13:28:53 -0700
-

Alan Smaill says...

george <greeneg@xxxxxxxxxx> writes:

On Aug 20, 11:39 am, stevendaryl3...@xxxxxxxxxx (Daryl McCullough) wrote:

Bell's Theorem proves that no measurable function f can satisfy this constraint. However, Pitowsky proved that if one assumes the continuum hypothesis, one can construct a nonmeasurable function that satisfies this constraint.

One line of the truth table still has not been completed here. If one DENIES the continuum hypothesis, can there still exist a NON-constructible nonmeasurable function that satisfies the constraint? Or is the truth of the CH necessary to the existence of the non-measurable function at all (regardless of whether it can be proven to exist)?

Since the proof given uses Martin's Axiom and not the stronger CH, and ZF+MA is consistent with not AC (assuming ZF consistent, presumably), the existence of such functions is consistent with not CH.

As I understand Pitowsky's construction, what is needed for the construction to go through is something along the lines of this axiom:

A: There exists a well-ordering $<$ of the reals such that for every real x , the set of all $y < x$ has Lebesgue measure 0.

The continuum hypothesis of course implies this (because if you well-order the reals with order type ω_1 , then the

Re: Continuum hypothesis

set of all $y < x$ will always be a countable set, which always has Lebesgue measure zero). But my axiom A doesn't imply the continuum hypothesis.

--
Daryl McCullough
Ithaca, NY