Re: The shocking truth about the naturals

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- *From*: Boris Borcic <bborcic@xxxxxxxxx>
- *Date*: Fri, 07 Sep 2007 19:33:53 +0200

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george wrote:

> On Sep 6, 9:31 pm, Boris Borcic <bbor...@xxxxxxxxx> wrote:
>
> > > ng as this old argument is, it is curious it has
> > > escaped the determined skeptic that the upward
> > > Löwenheim−Skolem
> > > theorem incontrovertibly shows we can't really prove any
> > > collection
> > > countable.
> >
> > Since the above technically alludes to me (as current last representative), I guess it is legitimate for me to remark on it... that (1) my /own/ use of "incontrovertibly" indeed alluded to the unavoidability of "determined skeptics"
> > and further adepts of the church of univocity, and (2) that my alluding to the necessity of counting with them is not sufficient cause to count myself among these groups.
>
> I am a little surprised that AK didn't give you credit for bringing
> this up.
> It is true that it is an intellectual point in its own right,
> regardless of who
> said it last, but since I wasn't reading the "physical process"
> thread, I
> didn't realize that you had inspired him.

Well, thanks for caring, the real point for me is not one of priority but one of community, in the sense that it was an issue I spotted silently some 25 years ago while browsing on my own through the "fundations" and "set theory" shelves of some math library. A confirmation that the issue wasn't just a private invention was deserved, and AK's reluctance to provide me with any... was none too welcome. Your own intervention, when I finally asked Google about "Wittgenstein+Loewenheim+Skolem" to discover this thread by way of the sci.logic archives, entailed the nice discovery that my assessment was correct.

> The relevant point here (which is converging with the point of my long
> thing
> from Sept.6) is that the situations are INHERENTLY not symmetric.
> "Prove" does not have just one meaning. Around here, we are
> artificially
> RESTRICTING the meaning of prove to "prove in classical finitary 1st−
> order logic".
> This paradigm is founded on the concept of a FIRST−ORDER LANGUAGE.
> This language (or method of language−formation, founded on simple
> "signatures") HAS A *DEFINITION*.

I think I understand that. Proofs are relative to a theoretical frame. And languages having definitions is not a mystery for an applications programmer. I'll re−read your sept.6 piece with interest. I am not sure I understand where you locate the dissymmetry, because I don't understand (yet) how the restriction to "prove in classical finitary 1st−order logic" creates a(n artifical) symmetry (what you appear to be saying). Instinctively it appears clear that the downward theorem impacts on interpreting the upward theorem in a way that has no converse (but, well, instincts...).

And I do not quite understand how the proof for the upward theorem (as sketched in the wikipedia) could be said to cather to "classical finitary 1st order logic" just as well as does the downward proof (since the former enrolls non−countably many first−order predications of differences between non−countably many constants; or else it is the case that I don't understand the intent of "finitary" − not to speak of the proof itself).

Notwithstanding the above; and while the downward and upward proofs (wikipedia sketches) do look quite different, I am wondering − more as an artistic exercize than anything else − how far could succeed an effort to formulate a simultaneous proof of both the upward and the downward theorem(s), given that their statements show lots of common structure.

Regards, Boris Borcic

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