Re: Cantor's definition of set

Source: http://sci.tech−archive.net/Archive/sci.logic/2007−10/msg00912.html

- *From*: G. Frege <nomail@invalid>
- *Date*: Fri, 26 Oct 2007 23:39:54 +0200

On Fri, 26 Oct 2007 13:29:08 −0700, John Jones <jonescardiff@xxxxxxx> wrote:

> I'm fine with Platonic numbers [...]

Please let's keep that in mind, since it simplifies the following discussion very much! So −for the sake of the argument− let's assume that there _are_ some mathematical objects called (natural) numbers.

Now

> 1,2,3,4,5... is often portrayed as numbers. But aren't they examples of the signs we use to portray numbers, ...

You are mixing op names with the objects denoted by this names. (−−> use−mention distinction!)

In the following I'm talking about certain numbers, namely the numbers 0 and 1:

The natural number 1 is the successor of the natural number 0.

I do this by using names for the mentioned numbers: the names "0" and "1".

"In the sentence the name represents the object.

Objects I can only /name/. Signs represent them.
I can only speak /of/ them. I cannot /assert them/.
[...]"

(L. Wittgenstein, Tractatus Logico−Philosophicus, 1921)

> and aren't these signs simply arranged in a sequence ...

Yes, if I consider the sentence "Let's consider the numbers 0, 1, 2, 3, ... " Then in this sentence we see a sequence of names: "0", "1", "2", "3" (in this order).

Moreover, the natural numbers actually ARE ordered in a certain "natural" way. 0 usually is considered the first natural number. 1 is considered to be the successor of 0, 2 is considered to be the successor of 1, etc. (Note that we are talking about numbers here, not about signs; but we do that using certain signs/names, of course.)

> and not numbers after all?

"In the sentence the name represents the object.

Objects I can only /name/. Signs represent them.
I can only speak /of/ them. I cannot /assert them/.
[...]"

(L. Wittgenstein, Tractatus Logico−Philosophicus, 1921)

This is true for mathematical objects too. (Here the "realistic picture" of mathematical objects is rather helpful. −−> Mathematical Platonsim.)

> For I cannot use 1,2,3,4 ... mathematically.

Sure you can.

1 + 2 = 3.

That's certainly a true claim. Or with other words, the sum of the two numbers 1 and 2 equals 3.

> 1,2,3,4 ... does not occur in any mathematical calculation.

Well, if you say so. Of course, the NUMBERS 1, 2, 3 do not "occur" in the /statement/ "1 + 2 = 3". On the other hand the names "1", "2". "3" used in the statement "1 + 2 = 3" certainly refer to the numbers 1, 2 and 3.

> What I am saying is, you can't pull a number out of the application
> that generates it.

If you say so. But so what?

> It would seem, if this is true, that a set of numbers is an impossibility.

If you say so. But even the "antecedence" of this conditional is doubtful. Moreover a set of natural number certainly is not doubtful (if there actually are such objects as numbers and sets.)

So I'll just repeat myself:

~~~~~~

Let's consider Cantors "definition" again:

"By a 'set' we mean any collection M into a whole of definite,
distinct objects m (which are called the 'elements' of M) of
our perception [Anschauung] or of our thought."

So if we assume that there actually are natural numbers

1, 2, 3, 4, 5, ...

(say in some Platonic sense) we might consider the collection into a whole of the numbers

1, 2 and 3

(which are definite distinct objects).

Hence let M be the collection containing exactly the numbers

1, 2, and 3.

In symbols:

M = {1, 2, 3}.

Where's the problem?

~~~~~~

So at least *I* can't see any _mathematical_ problems here.

> The second major problem which simply won't go away, is this: A set is the concept of a particular collection or group.

Maybe it would be better to say: A set is a particular collection ...

> At least I have seen a set described as either a collection or a group.

Right.

> Now a collection does not support sequence

− at least it seems...

But actually it does, as can be shown!

> ...in fact if a collection could be a sequence, it would be a sequence and not a collection.

Non sequitur. Actually, in axiomatic set theory certain sets are called /sequents/ (and some are even called /functions/, etc.).

The following might be helpful for you:

"/Set−theoretic reductionism/ is the view that all the abstract objects that are talked about in mathematics can be represented as sets. These representations are called the /set−theoretic surrogates/ for the mathematical objects in question. Perhaps the best−known example would be taking the /finite von Neumann ordinals/ as the set−theoretic surrogates for the natural numbers. [...] When, however, one persists in thinking or talking about mathematical objects without conceiving of them as their set−theoretic surrogates, one is said to be thinking or talking about those objects /sui generis/."

(Neil Tennant, A brief account of the fundamentals of set theory, 2003)

Another voice:

"Our contemporary orthodoxy: To show that there are so−and−sos
is to prove 'So−and−sos exist' from the axioms of set theory."

(Penelope Maddy, Mathematical Existence)

> Don't get me wrong here. I can have a set of sequences, but the the
> set itself, on its own merits, cannot support a sequence.

It can. Believe me. :−)

> I must establish the presence of a sequence independently of its
> membership in a set.

No. If you are interested, I'll show you how to perform this task.

> This, I advance, is another reason why I cannot have a set of numbers.

Wrong conclusions from false assumptions. :−)

F.

−−

E−mail: info<at>simple−line<dot>de
.