

Re: Zuhair's set theory: Corrected

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Hi all,

Zh is the set of all sentences entailed by (from classical FOL with identity) the following non logical axioms:

- 1) Axiom of sethood: $\exists x \exists y: x \in y$
- 2) Axiom of Extensionality: $\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y$
- 3) Axiom of Transitive closure:

$\exists x \exists y (\exists m (m \in x \rightarrow m \in y) \ \& \ y \text{ is transitive } \&$

$\forall m ((m \in y \ \& \ \sim m \in x) \rightarrow \exists z (z \in y \ \& \ m \in z)) \ \&$

$\forall k ((k \text{ subset_of } y \ \& \ \exists m ((m \in k \ \& \ \sim m \in x) \rightarrow \exists z (z \in k \ \& \ m \in z)) \ \& \ \sim k = 0) \rightarrow \exists m (m \in k \ \& \ m \in x))$).

Definition: $y = Tc(x) \leftrightarrow (\exists m (m \in x \rightarrow m \in y) \ \& \ y \text{ is transitive } \&$

$\forall m ((m \in y \ \& \ \sim m \in x) \rightarrow \exists z (z \in y \ \& \ m \in z)) \ \&$

$\forall k ((k \text{ subset_of } y \ \& \ \exists m ((m \in k \ \& \ \sim m \in x) \rightarrow \exists z (z \in k \ \& \ m \in z)) \ \& \ \sim k = 0) \rightarrow \exists m (m \in k \ \& \ m \in x))$).

- 4) Axiom of Uniformity: $\forall x: \sim x \in Tc(x)$

- 5) Axiom schema of Comprehension: if Q is a formula in which x is not free, then all closures of

$\exists x (\forall y (y \in x \leftrightarrow Q(y)) \leftrightarrow \forall y (x \in Tc(y) \rightarrow \sim Q(y)))$

are axioms.

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Theory definition finished.

Theorems:

$\exists x \forall y \sim y \in x$

Let $Q \leftrightarrow \sim y = y$

Then it is clear that $\forall y (x \in T_c(y) \rightarrow y = y)$ is a true statement.

Then according to comprehension $\forall y (y \in x \leftrightarrow \sim y \in y)$ is true

Thus $\exists x \forall y \sim y \in x$

From extensionality we prove that $\exists! x \forall y \sim y \in x$

and we define this x as 0 .

Now this theory avoid Russell's paradox easily.

Let $Q \leftrightarrow \sim y \in y$

Then $\forall y (x \in T_c(y) \rightarrow \sim y \in y)$ is always false according to this theory

and of course $\forall y (y \in x \leftrightarrow \sim y \in y)$ is false from Russells paradox

Thus from comprehension we have $\exists x (\text{False} \leftrightarrow \text{False})$

or simply $\exists x (\text{true statement})$ which is true since we proved that 0 exist.

In a similar way Cantor's paradox is solved since if $Q \leftrightarrow y = y$

we will have both statments $\forall y (y \in x \leftrightarrow y = y)$ and $\forall y (x \in T_c(y) \rightarrow \sim y = y)$

as false statements in this theory.

Regarding the set of all ordinals this will lead to $\forall y (y \in x \leftrightarrow y \text{ is ordinal})$

which is clearly false in this theory because it would be in itself and thus contradicting its own definition, since ordinals are Regular sets by their own definition, Accordingly by comprehension this leads to falsifying the second statment that is $\forall y (x \in T_c(y) \rightarrow \sim x \text{ is ordinal})$ since x would be in itself and thus there is ordinal that contain it.

Axiom of pairing, union, power, separation schema, replacement schema all can be proved in the same manner, Infinity also can be proved.

Zuhair

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