Re: The empty set

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* *From*: MoeBlee <jazzmobe@xxxxxxxxxxx>
* *Date*: Sun, 16 Dec 2007 14:38:57 −0800 (PST)

Two of my replies to apoorv, are not displaying in a certain news interface. Also, one of his replies is not displaying, so I don't know what is in it for me to reply. Here are my two posts again:

On Dec 14, 10:41 am, apoorv <sudhir...@xxxxxxxxxxx> wrote:

> On Dec 14, 6:00 am, G. Frege <nomail@invalid> wrote:
>
>> Actually, we often use the definition
>>
>> 0 =df {x : x =/= x}.
>
> How do we know that this defines a set ?

Because we prove E!yAx(xey <−> ~x=x)

For example, we know that

> V= df {x:x=x} is not a set.

And in that case we DON'T have a proof of

E!yAx(xey <−> x=x)

> Since in ZFC, all variables can only instantiate to sets,
> Ax(x!e0), means that no set belongs to 0; could 0 not contain non
> sets, such
> as V or possibly itself?

No, since in ZFC we can prove:

Ax x is set

where 'x is a set' is defined by:

x is a set <−> (x is a class & Ez xez)

and

x is a class <−> (x=0 v Ez zex)

I.e., in Z set theory we can prove that every object is either 0 or is
an object that as members and is itself a member.

(There's a redundancy in that definition for Z set theories, but I
leave the redundancy so that the definition is portable to other
theories in which the definition does not have the redundancy.)

> As you point out, A is necessary for the definition in ZFC, but
> otherwise
> completely irrelevant.

The symbol 'A' is NOT necessary to define '0'.

All that is needed to define '0' is:

Theorem: E!yAx ~xey

Definition: 0=y <−> Ax ~xey

> At least, in this one case, the use of separation invariably seems
> to amount to the use of 'unrestricted comprehension'. Is that
> permissible?

No. There is no unrestricted comprehension schema in Z set theories.

> Just one more thought. Sets are treated as undefined.

Actually, the predicate 'is a set' can be defined, just as I did
above.

> But,
> intuitively, a set is something that contains something.

If that is the common non−mathematical intuition, then mathematics departs with the common intuition at that point. If that bothers you, then consider the mathematical notion to be described by the word 'zet' instead of 'set'. Doing so won't have any substantive affect on the formal theories.

> If something contains nothing, would that something be a set?

As mathematics defines 'set' and 'the empty set' (or '0'), the empty set is a set.

Again, if that troubles you, then just read 'zet' wherever the word 'set' appears in exposition of the formal theory.

I never understand why you don't just take a look at the axioms and the proofs from them.

> From the axiom schema of separation and axiom of extensionality, we

get:

E!yAx ~xey

Then we define:

0=y <−> Ax ~xey

That's all that we need to define '0' and without useless quibbles about whether a set can be empty or not.

MoeBlee

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On Dec 14, 10:55 am, apoorv <sudhir...@xxxxxxxxxxx> wrote:

> If there is no universal set (containing all objects of the theory),
> then
> how can we have 0 as the absolute complement of the non existing
> 'universal set'?

We DON'T.

> If we take our universe as the class of all sets,

What does "our universe" mean?

> then would 0 be the
> class of all
> 'non sets'?

If we are in a CLASS theory such as Bernays theory, then there is the class of all sets but it has no complement, so, no, 0 is not the class of all non−sets, even in ordinary class theories.

> There is at least one non set, namely, the class of all
> sets.

In a class theory, that is correct.

MoeBlee
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