Re: T−relevant logic

Source: http://sci.tech−archive.net/Archive/sci.logic/2008−01/msg00013.html

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- *Date*: Mon, 31 Dec 2007 22:54:56 −0800 (PST)

On Jan 1, 8:03 am, Newberry <newberr...@xxxxxxxxx> wrote:

> We need to replace the classical logic with truth−relevant logic (see below.) Here is why.
> Let us consider the sentence "There are no round squares":
>
> ~(Ex)(Sx & Rx)   (1)
>
> If there are no round squares, to what are we then attributing non−existence? Do round squares subsist in order to enable us to talk about them even though they do not exist? Certainly not. The sentence above can be interpreted as " 'round square' does not have a denotatum."
>
> We can also interpret (1) as "all squares are non−round."
>
> (x)(Sx −−> ~Rx)   (2)
>
> But 'round square' still does not denote anything.
>
> What about "All round squares are large"?
>
> (x)((Sx & Rx) −−> Lx)   (3)
>
> Since 'round square' does not denote anything, (3) has exactly the same meaning as "All ghudalbirgs are large", that is, it has no meaning.
>
> Can (3) be interpreted as
>
> ~(Ex)((Sx & Rx) & ~Lx)   (4) ?
>
> That is, can we say then that " 'small round square' does not have a denotatum"? Perhaps, but (3) can also be interpreted as attempting to attribute a property to round squares.
>
> We need a logic where
>
> ~(Ex)(Px & ~Px)   (5)

is derivable, but

(x)((Px & ~Px) −−> Qx)   (6)

is not. What kind of logic will accomplish this? We will not be able to derive (6) if

P & ~P −−> Q   (7)

is not derivable. (7) is indeed notoriously counterintuitive − it is the paradox of material implication. This is a further indication that we are proceeding in the right direction.

In the logic NAFL, P&~P−−>Q is theorem of a consistent theory T in which either P or ~P is provable. If P is undecidable in T (i.e., neither P nor ~P is provable in T), then P&~P−−>Q is not provable in T. This makes perfect sense in NAFL, where P&~P−−>Q is equivalent to Pv~PvQ, and Pv~P is provable in T when either P or ~P is provable (and not otherwise). Note that there is absolutely nothing counter−intuitive about this.

Since P&~P−−>O is not provable in a consistent NAFL theory T in which P is undecidable, there must exist a model for T in which P&~P is the case, but an arbitrary proposition will not turn out true in that model. At first signt the truth of P&~P in the said non−classical model appears counter−intuitive, but it has a perfectly valid explanantion. See the sci.logic thread "FOL/Inuitionistic logic versus NAFL. Part 1. Failure of non−contradiction" for explanations:

http://groups.google.co.bw/group/sci.logic/browse_thread/thread/48894ac0f1d11787/f34781b18deff9c4?#f34781b18de

We have two options:
1) P −−> Q # ~P v Q = ~(P & ~Q)
2) P −−> Q = ~P v Q = ~(P & ~Q)
Several logics were proposed along the lines of 1)
* Lewis's system of strict implication
* Ackermann's system of strenge Implikation
* Church's system of weak implication
* Various systems of Anderson and Bellnap

There are two systems of logic in the second category
* truth−relevant logic
* occurrence−relevant logic
There are minor differences between the two.

None of the system in the first category achieved a general acceptance. Nobody was able to provide a satisfactory interpretation of any of them. Richard Diaz developed t−relevant and o−relevant logics as a result of his criticism of the systems of Anderson and Belnap. We will use them for a different purpose.

In t−relevant logic NONE of the formulas below are derivable

(P & ~P) −−> Q (8)
(~P v P) v Q (9)
~((P & ~P) & ~Q) (10)

and in our interpretation they are all meaningless. By generalization we also find that NONE of

(x)((Px & ~Px) −−> Qx) (11)
(x)((~Px v Px) v Qx) (12)
~(Ex)((Px & ~Px) & ~Qx) (13)

are derivable and they are all meaningless.

Let Pxy stand for x is the proof of y, let Qx be satisfied by only one x = m. And let the Goedel number of

~(Ex)(Ey)(Pxy & Qy) (14)

be m. Then (14) is a Goedel formula. We then observe that if

~(Ex)Pxm (15)

then (14) is meaningless analogically to (13). This is in accordance with our expectations; we came to a similar conclusion in another thread.http://groups.google.com/group/sci.logic/browse_frm/thread/e295044b4d... This gives us the basic insight that in all likelihood t−relevant logic is the way to go.

The way forward is NAFL. I will open a long−pending thread on NAFL now that I have cleared off my year−end commitments at IBM. I can always hope that in this new year, someone somewhere in the academic community will recognize NAFL's worth and be honest enough and straightforward enough to acknowledge it.

Regards, RS

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