Re: Torkel Franzen on truth

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- *From*: MoeBlee <jazzmobe@xxxxxxxxxxx>
- *Date*: Fri, 4 Jan 2008 16:16:48 −0800 (PST)

On Jan 4, 2:44 pm, "Nam D. Nguyen" <namducngu...@xxxxxxx> wrote:

> For what it's worth, I actually don't use a different definition of the word "model" here, in FOL context. The phrase that seems difficult to some to understand is:
>
> (1) 'true relative to a model'
>
> Now suppose you believe PA is consistent and let M be a model of PA, then the following meta level statement would be true:
>
> (2) '2=1+1' is true in M.

We don't need 'belive' there. We could just say:

If M is a model of PA, then '2=1+1' is true in M.

> The question though, if we change the inference rules and come up with a different reasoning framework, say (FOL)', then would (2) still be true?

Yes.

The inference rules don't alter what sentences are true in what models.

What the inference rules change is what theories we get.

First order PA is DEFINED to be the set of sentences that are entailed in classical first order logic from the first order PA axioms.

With a set of inference rules that doesn't yield the same theorems as classical first order logic you get a DIFFERENT theory from PA, you get HA (if the rules are intuitionistic logic) or whatever you want to name each DIFFERENT theory depending on a different logic.

> The answer is "Not necessarily", depending on what changes of
> rules of inference that have taken place of course. Now if we consider
> the formal system PA as just a *collection of axioms* then syntactically
> PA is the same.

But we DON'T consider PA as just a collection of axioms. We consider PA to be the set of sentences entailed by that collection of axioms. (Or some people say it is the pair <A T> where A is the collection of axioms and T is the set of sentences entailed by A.)

> But what we consider as M might or might not be an absolute
> model of PA, right?

What does "absolute model" mean?

> In that context then, '2=1+1' is being true in M is
> relative to the fact M might or might not be a model, depending on who's
> doing the reasoning. In that context, then the following statement
> would make sense:

I'd grant that it depends on how we're doing the reasoning in the META−theory, since 'true in the model M' is defined in a meta−theory for first order PA and our proofs of whether something is true in the model are either done in the meta−theory or we rely upon the soundness theorem (which also is proven in the meta−theory) to infer that what is proven in the object theory is true in every model of the axioms of that object theory, as well as it is in the meta−theory that we prove that axioms are true in whatever models we claim the axioms to be true in.

But that's not what you're talking about. Your present line of argument is, as usual, confused and ill−premised.

> (3) '2=1+1' is true, relative to a model.
>
> which is not different in nature from the statement:
>
> (3') the speed of the train is 100 km/hour, relative to the framework M.
>
> At any rate, '2=1+1' is no more absolute than 'the speed of the train is
> 100 km/hour'.
>
> Those who believes otherwise would not realize the relativity nature of

> human mathematical reasoning (through FOL at least).
>
> Now there is another "more technical" way to demonstrate the model−relativity connoted in (3), and I did allude to this way a couple of times in the past. Basically, if we consider a FOL system sPA ("super" PA) whose languages contains *infinite* symbols:
>
> L(0, S,+,*,<, S',+',*',<', S",+",*",<", S''',+''',*''',<''', ...)
>
> In other words, besides symbol 0, the rest would be grouped together and each group, when coupled with 0, would form a language we could use to formalize a theory we'd name "PA". [This alone would signify the relativity of "PA" and associated models − and (3). Wouldn't it?]

NO! That's silly! Just CALLING something 'PA' doesn't show any relativity other than the quite prosaic sense we all already know that if you say "Sally Fields played the role of James Bond" then that is true if by 'Sally Fields' you are referring to the person Sean Connery.

In some given overall mathematical context, such as a particular set theory to serve as a meta−theory, we DEFINE PA to be a certain exact mathematical object: a certain exact set of finite strings of symbols. Once we make that definition, it's just silly to worry about what happens if you say, "Oh, we get something different if we use 'PA' to stand for some other thing."

> The (infinite number of) axioms of sPA would be the union of those individual axioms per each ("PA") group mentioned above.
>
> Now let's examine the anatomy of a model M of a general FOL formal system T. In a nutshell, the major components of M are:
>
> c1: a set S of individuals of which certain n−ary relations would exist.

Okay, a non−empty set.

> c2: a collection of n−ary relations, each of which would correspond to an n−ary symbol of the language.

Hmm, I suppose that's okay. But I'd rather say, a function that assigns to each n−ary predicate symbol of the language an n−ary relation on S.

> c3: collection of (subjective) interpretations, each of which would predicate
> a theorem−formula as true.

YOU want c3 for whatever odd reason you do. I have no use for c3.
Through c2 you had a nicely rigorous mathematical definition going,
but then you obliterated it with the UNDEFINED terminology "subjective
interpretations" and "would predicate a theorem−formula as true".

> Now MoeBlee suggested above:
>
> "Anyone may state definitions and explicate a discussion on the basis
> of those definitions."

Sure. Though that was not intended to disallow that we may find
certain definitions and explications to clash so strongly with
ordinary terminology as to be irritating to work with.

> So in this context here and now, let's call S a "structure" (and temporarily
> forget if some text books reserve this word for something else).

So, a 'structure' is defined by you now to be what we ordinarily call
'the universe' or 'the domain of discourse' of a structure.

What is the advantage of you so confusingly switching these
defintions?

> Then it's
> not hard to see that the relativity of a model M would come from component c3!

Since c3 is UNDEFINED NONSENSE, it's hard to see what comes from it!

> For example, if the formula is "a < b", S = {a',b'}, and the n−ary relation
> is {(a',b')}, I still could at my subjective willingness interpret (or predicate)
> "a < b" as false while you or any other as true.

So who the L cares about that?

We already have a formal mathematical definition of 'true in the
model', but you'd rather we use some junky UNDEFINED nonsense about
"subjective interpretation" instead. What possible advantage comes
from that?

> And so relative to whose
> predicating or interpreting, M would be or *not* be a model of say T = {a < b},
> or for that matter of T = {~(a < b)}.
>
> Now back to sPA, let S be the structure (i.e. the _set_) of individuals of a
> model of the integers (i.e. not the natural numbers). The long and the short
> of it is out of S, we know there exist _uncountably many_ "successor" functions
> S()'s, hence uncountably many n−aries "addition", "multiplication", and "less−than".
> Put if differently, not only S is a structure for sPA, it would be the very same
> structure for _uncountably many_ models of each "PA" theory (written in L(sPA)).
> But it's not hard to demonstrate that due to the subjective interpretation in c3,
> if one interpret S as a model of a "PA", others might disagree and (re)interpret S
> as *not* a model this "PA".

Forget about whatever involves c3. If you'd have me adopt your c3,
then I might as well have you adopt: A sentence is true in a model M
iff one of the snails in my garden leaves, on the walkway, a trail
more than 4 inches on the second Wednesday of the next month.

> Of course when we talk about "PA" we typically talk about it outside the context
> of this orangutan sPA system. But it should not matter! Given *any* L(PA),

No, there is no "any" L(PA). L is an OPERATION. For each theory T,
there is exactly ONE object that is L(T). For each theory, there is
exactly one object that is THE language of that theory.

> one could
> consider it as part of L(sPA).

No problem with L(sPA) as different from L(PA) if sPA and PA are
different and happen also to have different languages.

> And given a model − over a structure S − of *any*
> "PA" theory one could interpret this structure S as a non−model of this "PA".
>
> In summary, a model is always (at least) *relative* to:

In summary, apply c3, then check the snail trails each first Wednesday
of the month, then take the Boolen product, then toss a coin for an
arbitrary number of trials, then disregard having done all that, then
declare the "relativity of predicating", then take deep inhalations
from a bottle of model airplane glue and forget about everything
whatsoever.

MoeBlee
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