Re: Largest Set in ZFC?

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On Mar 5, 12:15 am, reaste...@xxxxxxxxx wrote:

> I was looking at the axioms of ZFC on Wikipedia:http://en.wikipedia.org/wiki/ZFC
>
> I was wondering which sets can be defined in ZFC.
>
> The axioms given on Wikipedia only postulate
> one set, the set guaranteed to exist by
> the Axiom of Inifinity.
>
> According to Wikipedia, the empty set can
> derived from the other axioms if any set exists.

In ZF, the existence of an empty set is derivable from the theorem schema of separation (which is derivable from the strong form of the axiom schema of replacement). Uniqueness is derived from the axiom of extensionality.

> Its probably simpler to note that AoI says the
> empty set exists and is a member of a set
> closed under the successor function.

As the axiom of infinity is actually stated, it does not itself declare that there exists a set that has no members. Rather, it uses the symbol '0'. That that symbol designates a unique set that has no members is derivable from the definition of '0' as based on the existence and uniqueness theorem derived from the theorem schema of separation and the axiom of extensionality.

> AoI also "defines" successor, since none of the
> other axioms actually define successor.

Wrong. The pairing theorem (derivable from the strong form of axiom schema of replacement and the power set axiom) and the union axiom provide for a successor of any set, and the axiom of extensionality for uniqueness. The definition of 'successor of S' is as you give it below. That definition is not itself part of the axiom of infinity:

> The successor of set S is defined as S U {S}.

> AoI assumes the union of two sets is a set.

Pairing and union do that. And extensionality for uniqueness.

> The Axiom of Union only says the elements
> of a set can be in a union. I guess with Pairing
> and Union I can define the union of set {S,{S}}.

No, the way to do it this:

xuy = U{x y}

> AoI also seems to assume if S is a set then {S}
> is also a set. I would be interested to know if this
> can be proven with axioms other than AoI.

The definition is:

{S} = {S S}

and, of course {S S} we get from pairing..

> Wikipedia's AoI says:
> There exists a set X such that the empty set
> is a member of X and whenever y is in X, so is S(y).
> (where S(y) is the successor of y.)
>
> Can I say the following about X?
>
> If x is an element of X then
> x is the empty set or
> x is the successor of y
> where y is a member of X.

No. X might have OTHER members.

> If so, then it is easy to prove omega is
> not a member of X since omega is
> not the successor of any member of X.
> (and omega is not the empty set).

X is not yet a DEFINED set. The variable 'X' at this point is just an existential instantiation.

The we prove there is a unique X such that X satisfies the axiom of infinity and is a subset of any set that satisfies the axiom of infinity. Then THAT UNIQUE X is defined as w.

> The Axiom Schema of Separation allows me
> to define subsets of X, but only a countable number
> of such subsets. This follows because a
> function must be finitely defined and there can
> only be a countable number of finite definitions.

Not functions, but rather formulas. The subsets are defined by formulas, and there are only countably many formulas.

> The Powerset axiom says all the subsets of X exist,
> even the "inseparable" ones.

I don't know what you mean by "inseparable' in this context. Anyway, the power set axiom makes no mention of any such thing as 'inseparable'

> These would be subsets
> of X that can't be defined in a finite number of symbols.

If there are such subsets of the given set, yes.

> WIthout the Powerset axiom, can there be any set
> in ZFC larger than the set guaranteed to exist by AoI?

Larger in the sense of greater cardinality?

And by "can there be?" do you mean "is it consistent that there are?" or do you mean "is it proven that there are?" (Ordinarily, we woud

take the former sense.) Anyway, as far as I can tell(?), the answer is
that it's consistent but not provable.

MoeBlee

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